

INTERMEDIATE CERTIFICATE EXAMINATION, 1974

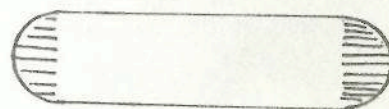
MATHEMATICS - HIGHER COURSE - PAPER 1
(300 marks)

MONDAY, 17 JUNE - MORNING, 9.30 to 12

SIX questions to be answered.
All questions are of equal value.
Mathematics Tables may be had from the Superintendent.

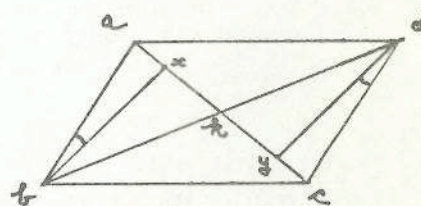
1. (a) Calculate $200 \times (0.875)^2 \sqrt{\frac{35}{18}}$ to two significant figures.
- (b) The scale on a map is 1 cm to 1 kilometre. What area in square metres is represented by 1 cm^2 ?
What area of the map represents 2.2 hectares?
[See Tables, page 5]

2. A solid piece of metal is in the shape of a cylinder with hemispherical ends of radius 3 cm, as in the diagram. When the hemispheres are removed, the volume of the piece is reduced by one-third.



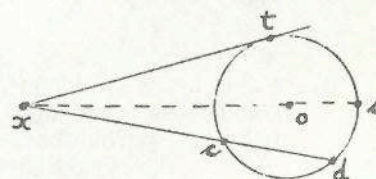
- (i) Find the total length of the original piece.
(ii) Find the total volume of the original piece, to the nearest cm^3 .
[Take 3.14 as an approximation for π . See Tables, page 7].

3. Prove that the diagonals of a parallelogram bisect each other.
The diagonals of a parallelogram intersect in k , as in the diagram. x, y are points on $[ac]$ such that $\angle abx = \angle cdy$. Prove that $bx dy$ is a parallelogram. By central symmetry, or otherwise, prove that the area of $\triangle abx =$ area of the $\triangle dyc$.



4. $[ab]$ and $[cd]$ are two chords of a circle which, when produced, intersect at the point x outside the circle. Prove $|xa| \cdot |xb| = |xc| \cdot |xd|$.

- (i) If xt is a tangent to the circle at t (see diagram), prove that $|xc| \cdot |xd| = |xt|^2$.
- (ii) If $|xc| = 4$, $|cd| = 5$, calculate $|xt|$.
- (iii) If $|to| = \sqrt{13}$, where o is the centre of the circle, find $|xs|$, where s is the intersection of xo and the circle as shown in the diagram.

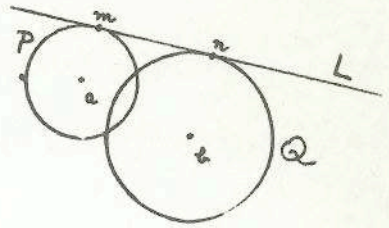


5. Prove that the medians of a triangle (i.e. the line segments joining the vertices of a triangle to the midpoints of the opposite sides) are concurrent and that they divide each other in the ratio 2 : 1.

abc is an isosceles triangle in which $|ab| = |ac| = 15 \text{ cm}$ and $|bc| = 24 \text{ cm}$. If g is the point of intersection of the medians, show that $|bg| = 3\sqrt{17} \text{ cm}$.

6. Explain how to construct a tangent to a circle at a point on the circle.

P and Q are two circles of different radii having centres at a and b , respectively. The line L is a tangent to P at m , and a tangent to Q at n (see diagram). Construct the image of P , Q and L by the reflection in the line ab and prove that the image of L is also a tangent to P and Q . If T is the image of L , prove that T and L intersect in the line ab .



7. a and b are two points $(-1, 2)$ and $(2, 3)$, respectively. Find (i) $|ab|$ (ii) the slope of ab (iii) the coordinates of c , the centre of (a, b) .

S_c and S_o are the central symmetries in the points c and o , respectively, where o is the origin. If d is the image of a by the composition of central symmetries $S_o \circ S_c$, find the coordinates of d . Show that $|ad| = 2|co|$ and that $ad \parallel co$.

8. L and K are two lines. The equation of L is $x + y + 2 = 0$; K contains the point $(1, -1)$ and $L \perp K$. Find the equation of K .

Find the co-ordinates of the vertices of the triangle formed by L , K and the x -axis.

Show that the centre of the circumcircle of this triangle is $(0, 0)$ and write down the equation of this circumcircle.

9. Using the same axes and the same scales, sketch the graphs of the functions $f : x \rightarrow \cos x$ and $g : x \rightarrow \sin 2x$ in the domain $0 \leq x \leq 2\pi$.

For what values of x is

(i) $\cos x = \frac{1}{2}$

(ii) $\cos x = \sin 2x$

(iii) $\sin 2x = 1$?

For what domain of values of x is $\sin 2x \geq \cos x$?

10. (a) Verify that for $A = 35^\circ$, $\cos 2A = 1 - 2(\sin A)^2$, correct to three significant figures.

(b) x and y are two lighthouses 21 kilometres apart. k and h are two ships such that k is collinear with the two lighthouses and such that $\angle hky = 70^\circ$. (See diagram) If $\angle xhk = 60^\circ$, $\angle yhk = 40^\circ$, calculate, correct to two significant figures, the distance between the two ships.

