

INTERMEDIATE CERTIFICATE EXAMINATION, 1973

 MATHEMATICS - HIGHER COURSE - PAPER II
 (300 marks)

 WEDNESDAY, 13 JUNE - MORNING, 9.30 to 12

Six questions to be attempted.

All questions are of equal value.

Mathematics tables may be obtained from the Superintendent.

1. (a) Calculate the compound interest on £1,200 for 2 years at 9% per annum.
 (b) A bought goods and sold them to B making a profit of 10%. B then sold the goods for £5.94 and his profit was 20%. What did A pay for the goods and what was B's profit in cash?

2. (a) Find the solution set of

(i) $2x - 1 < x - 2$ (ii) $2x - 1 < x - 2 \leq 3x + 10$

and graph the solution set on the number line in each case.

- (b) Factorise $2x^2 + 5x - 3$ and hence, or otherwise, find the solution set of $2x^2 + 5x \leq 3$ and graph the solution set on the number line.

3. (a) Use Venn diagrams to illustrate that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

- (b) A music group consists of 30 members each of whom can sing and also play an instrument. The group gives a number of performances and occasionally some members are absent. During each performance, however, 18 must play and 24 must sing, some playing and singing at the same time.

- (i) When 2 members are absent, how many sing only?

When no one is absent:-

- (ii) how many both play and sing at the same time?

- (iii) how many play only?

What is the greatest number:-

- (iv) that can play and sing at the same time?

- (v) that can be absent at the same time?

4. Let $P = \{a, b, c, d, e, f\}$. Draw a Venn diagram of P and put their names on the elements.

Graph the relations:-

(i) $R = \{(d, a), (d, f)\},$

(ii) $S = \{(f, a), (f, b), (f, c), (f, d)\}.$

Write down the elements of:-

(a) R^{-1}

(b) S^{-1}

(c) $S \circ R$

(d) $(S \circ R)^{-1}$

(e) $S^{-1} \circ R^{-1}.$

Is $(S \circ R)^{-1} = S^{-1} \circ R^{-1}$? Give a reason for your answer.

5. (a) The n th term of an arithmetic sequence is $2n + 5$. Write down the first four terms of the sequence and find which term of the sequence is 135.
 (b) The first three terms of an arithmetic sequence are 5, x , $10\frac{1}{2}$. Find x and the common difference of the sequence.
 (c) Write down the first four terms of the sequence $3(2)^{n-2}$ and say, giving a reason, whether the sequence is geometric or not.

6. Sketch the graph of the function f for $0 \leq x \leq 6$ given that:-

$$f(x) = -x^2 + 6x - 5.$$

Find from your graph the values of x for which

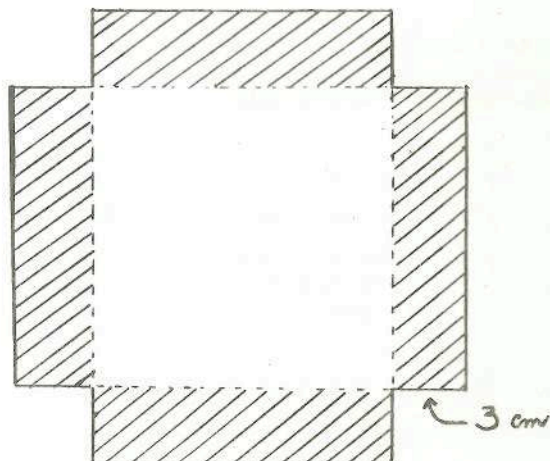
- (i) $f(x) \geq 0$,
- (ii) $f(x)$ is positive and increasing.

Show from your graph that $f(x)$ will never be equal to 5 and find those values of x for which $f(x) = x$.

7. An open tin box has a volume of 54 cm^3 . The box was made from a large square sheet of tin, A, by cutting away a small square of side 3 cm from each corner of A and then turning the four outer edges vertically upwards. (See the diagram in which the outer edges are indicated by shading). Find:-

- (i) the length of the side of the square A,
- (ii) the area of the square A.

(Note: you may leave your answer in surd form in each case).



8. (a) (i) Write the denary numbers 13 and $4\frac{1}{2}$ in binary form.

(ii) Write the binary numbers 1111 and 1.111 in denary form.

(b) (i) Find the value of each of the following:-

$$\log_3 9; \log_9 3; \log_3 \frac{1}{3}.$$

(ii) Find the solution set of:-

$$\{x \mid \log_{10} (x^2 + x + 8) = 1\}.$$

9. The following table shows the frequency distribution of the marks in an examination:-

| Mark | 0-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-100 |
|----------------------|------|-------|-------|-------|-------|-------|--------|
| number of candidates | 4 | 8 | 14 | 20 | 30 | 18 | 6 |

(Note: 0-20 means 0 but less than 20, 20-30 means 20 but less than 30 etc.)

Draw a histogram to illustrate the distribution. Find:-

- (i) the total number of candidates,
- (ii) the modal class,
- (iii) the median mark of the distribution.

10. Using the same axes and the same scales draw:-

(i) the line $2x + y = 0$,

(ii) the triangle abc whose vertices are $a(3,5)$, $b(2,2)$, $c(6,4)$.

If S is the set of points (x,y) of the triangle together with its interior, find the points of S for which:-

(iii) $2x+y$ is a minimum,

(iv) $2x+y$ is a maximum,

and find the minimum and maximum values.

Indicate the points of S for which:-

(v) $2x+y = 8$,

(vi) $8 \leq 2x+y \leq 12$.