AN ROINN OIDEACHAIS

INTERMEDIATE CERTIFICATE EXAMINATION, 1973

MATHEMATICS - HIGHER COURSE - PAPER I
(300 marks)

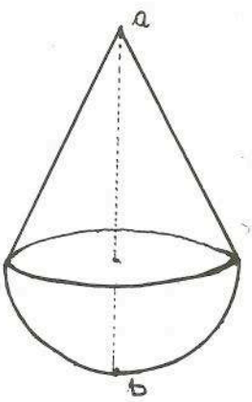
MONDAY, 11 JUNE - MORNING, 9.30 to 12

SIX questions to be answered.
All questions are of equal value.
Mathematics Tables may be had from the Superintendent.

1. (a) If $\frac{g}{l} = \left(\frac{2\pi}{t}\right)^2$, calculate the value of g correct to three significant figures, where $l = 155$ and $t = 2{\cdot}50$.

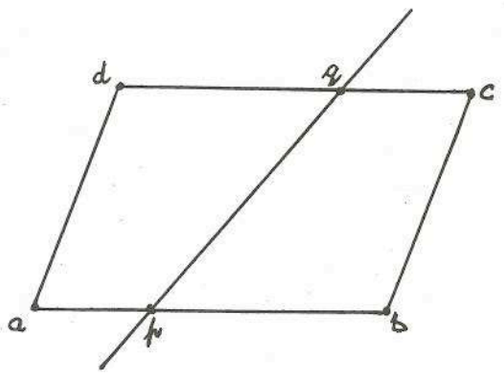
(b) A pond has a circular rim of length 2 kilometers. Calculate the area of the pond in hectares (ha). Give your answer to the nearest hectare (See tables, page 5).

2. A solid is composed of a hemisphere surmounted by a right circular cone, as in diagram, and the volume of the cone is equal to the volume of the hemisphere. Show that the height of the cone is equal to the diameter of its base. If $|ab|$, the total height of the solid, is 18 cm, find the volume of the solid correct to three significant figures.



3. Prove that any point on the perpendicular bisector of a line segment $[ab]$ is equidistant from a and b.

In the diagram $abcd$ is a parallelogram and D is a line intersecting $[ab]$ and $[cd]$ in p and q, respectively. Show how to construct D such that d is the image of b by reflection (axial symmetry) in D. Prove that the four sides of $dpbq$ are equal in length.

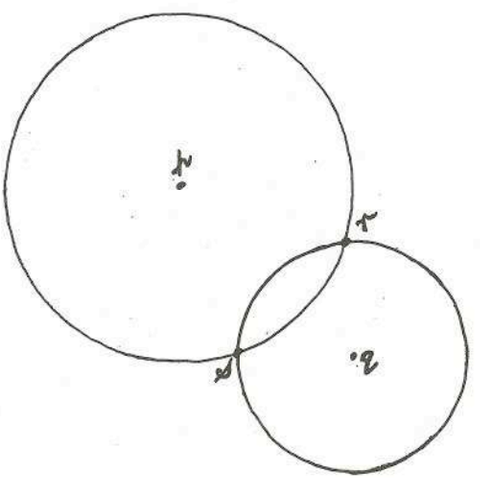


4. Show, with proof, how to construct the incircle of a given triangle.

The side of an equilateral triangle is of length 2 units. Show that the length of the radius of its incircle is $\frac{1}{\sqrt{3}}$ units.

5. (a) D is a line through the centre c of a circle K. What is the image of K by the reflection (axial symmetry) in D ? What is the image of K by the central symmetry in c ? Is it correct to conclude from your answers that reflection is equivalent to central symmetry ? Give your reasons.

(b) Two circles of centres p and q intersect in r and s, as in diagram. Prove that the line pq bisects the line segment $[rs]$ at right angles.



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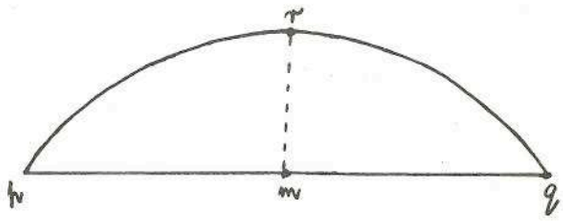
6. $[ab]$ and $[cd]$ are two chords of a circle that intersect in the point x, where x is not the centre of the circle. Prove that $|ax|.|xb| = |cx|.|xd|$.

prq is an arc of a circle, as in diagram.

$|pq| = 24$ cm, $|pm| = |mq|$, $rm \perp pq$, $|rm| = 8$ cm.

Calculate the length of the radius of the circle.



7. In the diagram a and b are the points (6, 2) and (-2, 6).

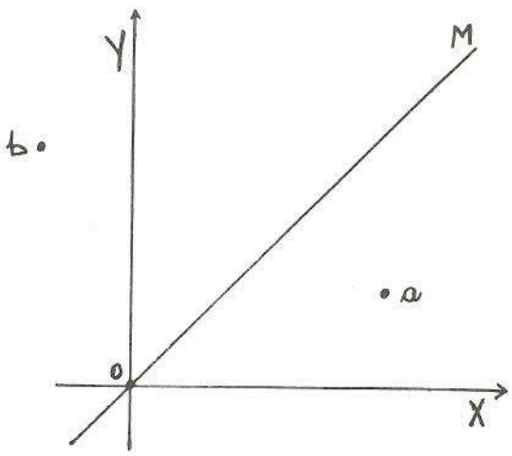
(i) Find $|ab|$.

(ii) Prove $oa \perp ob$.

M is the line whose equation is $y = x$.

If S_X, S_M are the reflections (axial symmetries) in the lines X and M, what are the images of the point a by S_M, S_X, $S_M \circ S_X$?

Show that the point b is the image of the point a by the reflection in the line $y = 2x$.



8. Find the equation of the circle with centre at the origin and radius of length $\sqrt{10}$.

Show that the line $x + 3y - 10 = 0$ is a tangent to this circle and give the coordinates of the point of contact.

9. Using the same axes and the same scales, sketch the graphs of the functions f and g in the interval $0 \leqslant x \leqslant 2\pi$, where

$$f : x \to \cos x, \qquad g : x \to -\cos x$$

Write down the range of each function.

Find from your graph

(i) the values of x for which $g(x) = f(x)$;

(ii) the maximum value of $f(x)$;

(iii) the minimum value of $g(x)$;

(iv) the maximum value of $g(x) - f(x)$.

10. (a) If $\angle A$ measures 55° and if sin B = 0·3420, B $\angle$ 90°, show that sin (A - B) $\neq$ sin A - sin B.

(b) A vertical mast is held in position by a taut wire connected to the ground at a and c and passing through a small ring firmly attached to the mast at b. A horizontal line through d, the foot of the mast, passes through a and c. If the distance between a and c is 10 metres and if $\angle bad$ and $\angle bcd$ measure 55° and 40°, respectively, calculate, to the nearest metre, the length of the wire. (See diagram).