INTERMEDIATE CERTIFICATE EXAMINATION, 1972

MATHEMATICS—HIGHER COURSE—PAPER I
(300 marks)

THURSDAY, 8th JUNE—MORNING, 9.45 to 12.15

Six questions to be answered.
All questions are of equal value.
Mathematical Tables may be had from the Superintendent.

1. (*a*) If $n = \dfrac{bc}{pq}$, find p, where

$$b = (7651)^{1/2},\ c = (0{\cdot}4073)^2,\ q = (2{\cdot}251)^3,\ n = 80{\cdot}53.$$

Give your answer correct to three significant figures.

(*b*) Calculate the value of h correct to two decimal places, given that

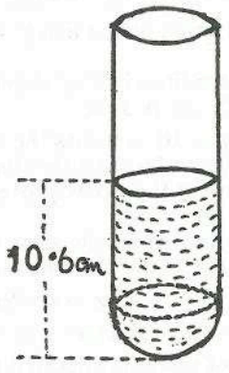
$$\frac{2}{h} = \frac{1}{a} + \frac{1}{b},$$

where

$$a = 23{\cdot}78,\ b = 0{\cdot}6487.$$

(See tables p. 26, 27).

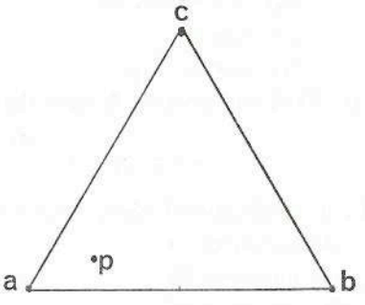
2. A tube in the shape of a cylinder with a hemispherical base contains water to a depth of 10·6 cm and its internal diameter is 4·8 cm (see diagram). Calculate (i) the volume of the water in cm³ correct to three significant figures, (ii) the total length of the tube if it is now half-full of water.



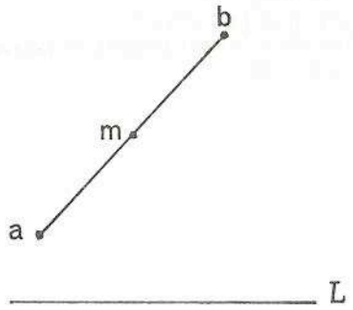
3. In the diagram p is any point inside the equilateral triangle abc. r_a and r_b are the rotations about centres a and b, respectively, such that r_a maps $[ac]$ onto $[ab]$ and r_b maps $[ba]$ onto $[bc]$. Construct the images of p by (i) r_a and (ii) r_b and show by a diagram that

$$r_a \circ r_b \neq r_b \circ r_a.$$

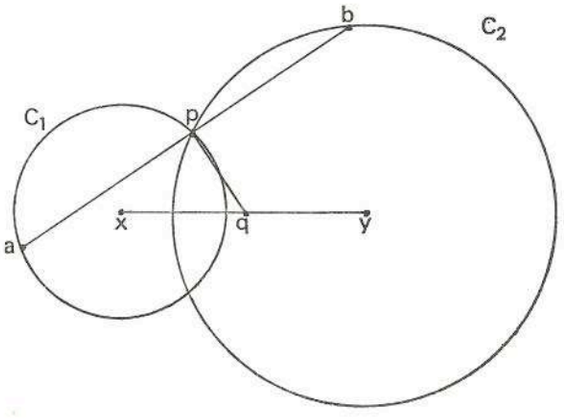
If r_c is the rotation about centre c which maps $[cb]$ onto $[ca]$, construct the image of $[ab]$ by $r_c \circ r_b \circ r_a$. Does your construction indicate that $r_c \circ r_b \circ r_a$ is a central symmetry? Give your reasons.



4. (i) m is the mid-point of $[ab]$ as in diagram. If a_1, b_1, m_1 are the images of a, b, m, respectively, by a parallel projection onto the line L, prove that m_1 is the mid-point of $[a_1, b_1]$.



(ii) C_1 and C_2 are two circles of unequal radii, as in diagram, their centres are x and y, as shown, and p is a point of intersection. The mid-point of $[xy]$ is q, $[ab] \perp pq$ and $p \in [ab]$. Prove that $|\,ap\,| = |\,pb\,|$.

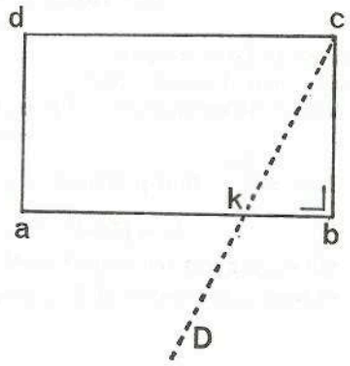


5. Illustrate by means of an example the meaning of the statement: "Equipollence is a transitive relation". Hence, or otherwise, prove that if a line contains the midpoint of one side of a triangle and is parallel to a second side it bisects the third side.

L, M, N are the perpendicular bisectors of the sides $[ab]$, $[ac]$, $[bc]$, respectively, of a triangle abc and $L \cap M \cap N = \{k\}$. If $\triangle abc$ has a right angle at a, prove that $k \in [bc]$.

6. Prove that the area of a triangle is equal to half the product of the base and the altitude (height).

In the diagram, $abcd$ represents a rectangular piece of paper and $|ab| = 12$, $|bc| = 9$. Find $|ac|$. If D is a line such that $[cb]$ is mapped into $[ca]$ (i.e. the image of $[cb]$ is a subset of $[ca]$) when the paper is folded in the line D, find the image of $\triangle cbk$. Hence, or otherwise, calculate the area of paper remaining when the triangular piece is removed.



7. If $\vec{t}$ is the translation mapping the origin on the point $(-2, 2)$, find the images by $\vec{t}$ of the points $a(2, -2)$, $b(4, -1)$, $c(6, 5)$. Hence, or otherwise, find the area of the triangle abc.

If S is the reflection in the line $y = x$, find the image of $(3, 4)$ by $S \circ \vec{t}$.

8. A and B are two lines having slopes m_1, m_2, respectively. Write down an equation in m_1 and m_2 when (i) $A \parallel B$, (ii) $A \perp B$.

The line $x - 3y + 10 = 0$ cuts the circle $x^2 + y^2 = 20$ at the points r and s. Find the coordinates of the mid-point of $[rs]$ and the distance of the mid-point from the centre of the circle. If k is the centre of the circle, prove that $\angle rks$ is a right angle.

9. Using the same axes and the same scales sketch the graphs of the functions g and h in the interval $0 \leqslant x \leqslant 2\pi$, where the functions are defined as

$$g: x \to \cos 2x (= y), \qquad h: x \to \cos^2 x (= y).$$

(i) Give the range (image) and period of each function.

(ii) In each of the following find the values of x from your graph for which

(a) $\cos 2x = \cos^2 x$.

(b) $\cos 2x = \frac{1}{2}$.

(c) $\cos^2 x = \frac{1}{2}$.

(iii) Find one interval K such that

$$x_1 < x_2 \Rightarrow \cos 2x_1 < \cos^2 x_2$$

for $x_1, x_2 \in K$.

10. In a quadrilateral $abcd$, $|ab| = 10$, $|bc| = 5$

$\angle abd$ measures $40°$

$\angle dab$ measures $55°$

$\angle dbc$ measures $60°$

(see diagram).

Find $|bd|$ and $|cd|$.

Give your answer correct to two significant figures in each case.