AN ROINN OIDEACHAIS

INTERMEDIATE CERTIFICATE EXAMINATION, 1971

MATHEMATICS — HIGHER COURSE — PAPER II
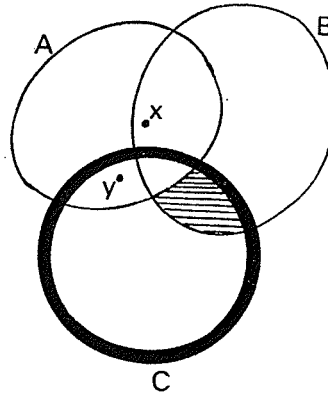(300 marks)

TUESDAY, 15th June — MORNING 9.30 to 12

Six questions to be attempted.
All questions are of equal value.
Mathematical tables may be obtained from the Superintendent.

1. Find correct to the nearest new penny the compound interest on £375 for 3 years at $7\frac{1}{2}\%$ per annum.

2. A, B and C are sets as in the diagram. Which of the following statements are true and which are false?

   (i) $x \in A \cap B$

   (ii) $x \notin B \cap C$.

   (iii) $x \notin (A \cap B) \cup C$.

   (iv) $A \cap B \not\subset A$.

   (v) $y \in (A \cap B)'$.

   (vi) $A \cap B \cap C \subset A \cup B \cup C$.

   (vii) $B \cap C \subset A$.

   (viii) $A \cap (B \cup C) \neq (A \cap B) \cup (A \cap C)$.

   (ix) $(A \cap B)' \subset A$.

   (x) $A \smallsetminus B = A \cap B'$.

   [Note: the symbol ' denotes the complement of a set, and shading denotes an empty set.]

3. (a) x is a number in the numberline such that $-2 \leqslant x \leqslant -1$

   Find (i) the greatest value,

   (ii) the least value,

   which each of these can have:

 $$1 - x; \quad x(1 - x); \quad \frac{1}{x}.$$

   (b) Factorise $2x^2 - 2x - 4$.

   Hence or otherwise find the solution set of
   $$2x^2 - 2x - 4 \leqslant 0$$
   and graph that set on the numberline.

4. (i) Graph the relation R defined by "is a divisor of" in the set B shown.
   Is R reflexive?
   Is R symmetric?
   Is R transitive?
   Also graph the relation S defined by "is 3 less than" and write down the couples of the relation $R \circ S$.
   Find a couple which belongs to $S \circ R$ but not to $R \circ S$.
   Is $R \circ S \subset R$? Explain your answer.
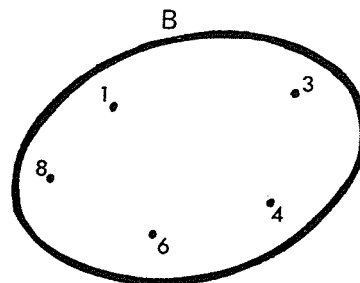
5. (a) The sum of the first five terms of an arithmetical progression is $26\frac{2}{3}$. The sum of the next five terms is 60. Find the first term and the common difference.

   (b) Write down the set of divisors of 352.

   n is a natural number and so is $\dfrac{352}{16 + n}$.

   Find the possible values of n.

[P.T.O.

6. (a) Examine the lines A, B and C in the graph. Then write down the equation of each.
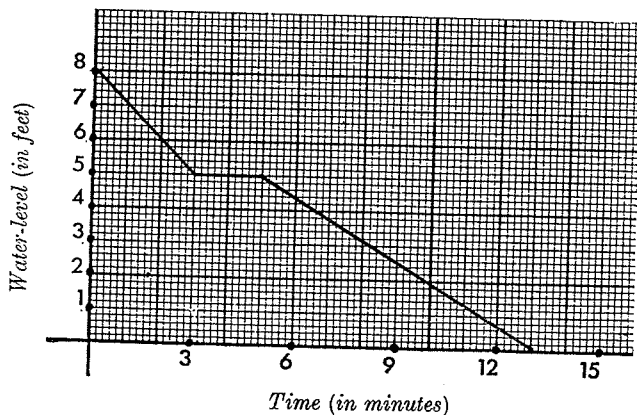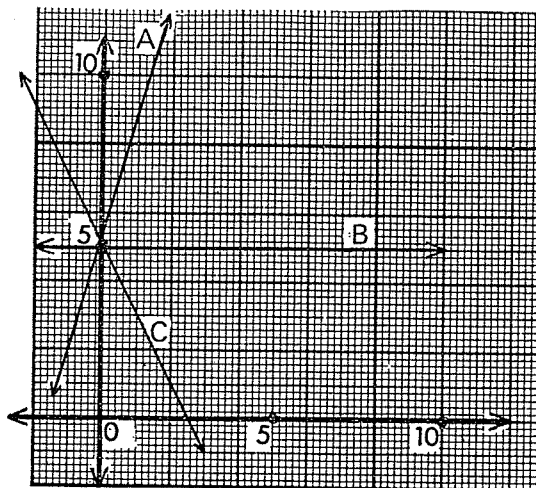
(b) An 8-feet-high cylindrical tank was full of water.
The graph on the left shows a record of the water-level at each minute as a result of opening, shutting and again opening a tap in the bottom of the tank.

Water-level (in feet)

Time (in minutes)

From the graph find:

(i) how many feet the water-level dropped in the first minute,

(ii) how much time passed before half the water had drained from the tank.

If the tank can hold 800 gallons compute the quantity of the water which drained off in the final minute.

7. (a) Write down a number which differs from $3\frac{1}{4}$ by less than one millionth.

(b) Find the least $n \in N$ for which $(0.3)^n < .001$.

(c) The three sides of a triangle were measured. Their lengths, given correct to the nearest tenth of a centimetre were 13·5 cm, 8·8 cm and 9·4 cm. Find the maximum and the minimum possible lengths of the perimeter.

8. (a) f is the function whose domain is $\{4, 3, 2, 1, 0, -1, -2\}$. The rule

$$f: x \to \sqrt{x^2 - 2} \quad \text{(i.e. } f(x) = \sqrt{x^2 - 2}\text{)}$$

tells how the image of each element in the domain can be found.

(i) Write down the set of images.

(ii) Which images are not real numbers?

(iii) Is $-f(2) = f(-2)$? Explain.

(b) Sketch the graph of the function g for $-3 \leqslant x \leqslant +3$ given that

$$g(x) = 4 - x^2$$

and find its maximum value.

9. A road-check on 40 motor vehicles yielded the following record of the number of occupants each carried:

3	1	2	1	4	1	2	2	5	1
1	3	1	3	3	6	2	2	1	1
1	4	1	3	1	6	2	2	1	5
4	4	5	1	3	4	32	1	5	6

Make out a frequency table and find the modal number of occupants. Find the arithmetic mean. Illustrate the data by a frequency polygon.

10. In the co-ordinated plane show the set K of points which simultaneously satisfy the inequalities:

$$2 \leqslant y \leqslant 4;$$
$$3x \geqslant 2;$$
$$2x + y \leqslant 10.$$

If (x, y) belongs to K find (i) the maximum value and (ii) the minimum value of $\frac{1}{2}x + y$.