

INTERMEDIATE CERTIFICATE EXAMINATION, 1971

MATHEMATICS — HIGHER COURSE — PAPER I
(300 marks)

WEDNESDAY, 9th JUNE — MORNING 9.45 to 12.15

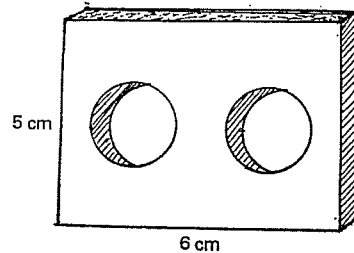
Six questions to be answered.

All questions are of equal value.

Mathematical Tables may be had from the Superintendent.

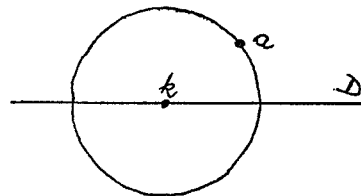
1. (a) A householder used 612 units of electricity in a given period. Find the total cost in pounds if 0.75p per unit is charged for the first 360 units, 0.65p per unit for the remaining units and if turnover tax is $2\frac{1}{2}\%$. Give your answer correct to three significant figures.
- (b) Given that 1 cubic centimeter of air weighs 0.00129 grams calculate the weight in kilogrammes of the air in a room which is 5.25 metres long, 4.5 metres wide and 3.2 metres high. Give your answer correct to one place of decimals. (1 kilogramme = 1000 grammes.)

2. A rectangular solid piece of metal weighs 500 grams and is 8 cm long, 5 cm wide and 5 cm thick. A technician cuts off a rectangular portion 6 cm long, 5 cm wide and $1\frac{1}{2}$ cm thick. Through this portion he drills two cylindrical holes of equal size so that finally he has a piece 100 grams weight. (See the diagram). Compute the diameter of each hole in centimetres to one place of decimals.

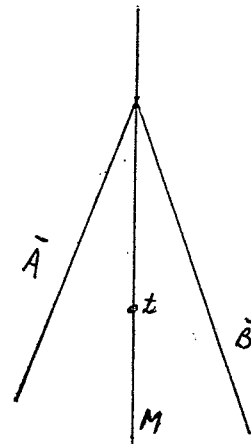


3. abc is a triangle and x, y , are the midpoints of $[ab]$ and $[ac]$, respectively. Prove that xy is parallel to bc . If $[xc] \cap [yb] = \{k\}$ prove that the areas of the triangles xkb and ykc are equal, and deduce that the area of the quadrilateral $axky$ is equal to the area of the $\triangle bkc$.
4. $abcd$ is a square. The images of the points a, b, c by S_d , the central symmetry in d , are x, y, z , respectively. Show that $xyzd$ is a square and use your diagram to write down four couples of S_d . Say which, if any, of the following statements is true and give your reason in each case:
- bd is an axis of symmetry of the set $\{x, y, z\}$
 - the image of $[ad]$ by the rotation $S_{ax} \circ S_{az}$ is $[ad]$, where S_{ax} is the reflection in ax and S_{az} is the reflection in az ,
 - $accz$ is a square.

5. (a) Prove that a circle maps onto itself by reflection in one of its diameters.
 a is a point of a circle as shown in the diagram, and k is the centre of the circle. Find the image of a by $S_k \circ S_D$ where S_k is the central symmetry in k and S_D is the reflection in D . Is the image of a by $S_k \circ S_D$ equal to the image of a by $S_D \circ S_k$? Give reasons.



- (b) If p is any point and Q is any line is it always true that $S_P \circ S_Q = S_Q \circ S_P$? Explain your answer by a diagram.



6. M is the bisector of the angle formed by the half-lines \vec{A}, \vec{B} as in diagram, and $t \in M$. Show that t is equidistant from \vec{A} and \vec{B} . Hence, or otherwise, deduce that the bisectors of the three angles of a triangle are concurrent.

7. The three points $a(1, 3)$, $b(4, 2)$ and $c(2, 1)$ form a triangle. The $\triangle a_1 b_1 c_1$ is the image of the $\triangle abc$ by the translation \vec{co} , where o is the origin. Find the co-ordinates of a_1, b_1, c_1 .
The $\triangle a_2 b_2 c_2$ is the image of the $\triangle a_1 b_1 c_1$ by S_o , the central symmetry in o . Find the co-ordinates of a_2, b_2, c_2 . Show that

$$S_o \circ \vec{co} \neq \vec{co} \circ S_o.$$

8. Show that the equation of a line containing the point (x_1, y_1) and having slope m is

$$y - y_1 = m(x - x_1).$$

Find the equation of the line L which contains the point $(3, 4)$ and which is parallel to the line $3x + 4y = 0$. Show also that L is a tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

9. (a) Use your tables to evaluate:

$$\sin 31^\circ 31', \quad \cos 17^\circ 47', \quad \tan 280^\circ$$

- (b) Sketch the graph of the function $f: x \rightarrow \sin x$ (i.e. $y = \sin x$) in the domain $-\pi \leq x \leq \pi$.
Using the same axes and the same scales sketch the graph of the function $g: x \rightarrow \sin 2x$
 $-\pi \leq x \leq \pi$.

Use your graphs to answer the following:

- (i) What is the period of each function?
 - (ii) For what values of x is $\sin x = \sin 2x$?
 - (iii) to what intervals does x belong when $\sin x > \sin 2x$?
10. g and h are two ships 50 miles apart and h is directly East of g . A distress signal from a trawler is picked up at g and h , and the trawler is then 32° East of South from g , and 9° West of South from h . Find to the nearest mile the distance from the trawler to the ship nearest it.