INTERMEDIATE CERTIFICATE EXAMINATION, 1970

MATHEMATICS — HIGHER COURSE — PAPER II
(300 marks)

MONDAY, 15th JUNE — MORNING 9.30 to 12

Six questions to be attempted.
All questions are of equal value.
Mathematical tables may be obtained from the Superintendent.

R is the set of real numbers.
$N_o = \{1, 2, 3, 4, \ldots .\}$

1. Calculate the Compound Interest on £1,070 for 3 years at $8\frac{1}{2}\%$ per annum. Give your answer in pounds correct to 2 places of decimals.

2. A manufacturer of golf balls gives a special price reduction of 10% on any orders valued at £15 or more. A customer finds that by making an order valued at £15 he can thus get 10 more balls for £15 than he could get by making 3 orders of value £5 each. Find the cost per ball.

3. (a) If $A = \{1, 2, 3\}$ graph the Cartesian product $A \times A$.
Write down a subset of $A \times A$ which is a function.

(b) f and g are two functions determined by the following equations:
$f(x) = x^2 - 3$ (i.e. $y = x^2 - 3$) $x \in R$.
$g(x) = 3 - x^2$ $x \in R$.
Graph roughly both f and g for values of x in the interval $[-2, 2]$.
f_{min} means the minimum value of f.
g_{max} means the maximum value of g.
Is $f_{min} = -g_{max}$? Explain your answer.

4. (a) Solve each of these equations:
(i) $x^2 + 1 = 17$ (ii) $x^2 + 1 = 9$ (iii) $x^2 + 1 = 1$ (iv) $x^2 + 1 = 0$.
Which equation has solutions which are irrational?
Which equation has solutions which are complex?

(b) On the numberline show the solution set of the inequality
$2x^2 + 3x + 2 < 7$.

5. (a) Write down the set $\mathscr{P}X$ whose elements are all the subsets of $X = \{a, b\}$.
Then (i) show that $\mathscr{P}X$ is closed under intersection, and
(ii) Say which element of $\mathscr{P}X$ is the identity with respect to intersection.

(b) Write down an equation which illustrates that Intersection is distributed over Union. Illustrate the situation by means of one or more Venn diagrams.

6. (a) Rewrite each of the equations below, replacing each x by the index (or exponent) which makes the equation true:
$\sqrt{6{\cdot}4} = (6{\cdot}4)^x$; $\sqrt[3]{48} = (48)^x$; $\frac{1}{\sqrt{13}} = (13)^x$.

(b) Solve each of the following equations:
$\log_2 8 = x$; $\log_4 8 = x$.
"If $\log_{10} x < 0$ then $x < 0$." Is this statement true?
Give a reason for your answer.

(c) Are these statements true? If not, rewrite each right hand side to make true statements.
(i) $\log_{10} a\ (b + c) = \log_{10} a + \log_{10} b + \log_{10} c$
(ii) $\log_{10} \frac{x}{a} = -\log_{10} \frac{a}{x}$

7. (a) a, ar, ar^2, . . . is a geometric sequence.
(i) Write down its nth term.
(ii) If $0 < r < 1$ prove that $ar^m > ar^{m+1}$ where m is any natural number.
(iii) If $a = 4$ and $r = 0{\cdot}1$ how many terms of the geometric sequence are greater than $0{\cdot}000000001$?

(b) If the population of a town has been increasing each year in such a way that the population at the end of each year is 2% greater than that at the beginning, what population would you forecast for the town on January 1st 1980 if its population on January 1st 1970 was 9,900? [Give your answer to the nearest 100.]

8. (a) The first four terms in an infinite sequence are $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$. Write down the nth term assuming that the observed pattern gives subsequent terms.

(b) Write down the nth term of any sequence which is an INCREASING sequence in which EACH TERM is less than 10.

(c) The nth term of a sequence is $\log_{10} n$, $n \in N_0$. Write down those terms of the sequence which are less than 1. Is the sequence increasing or decreasing?

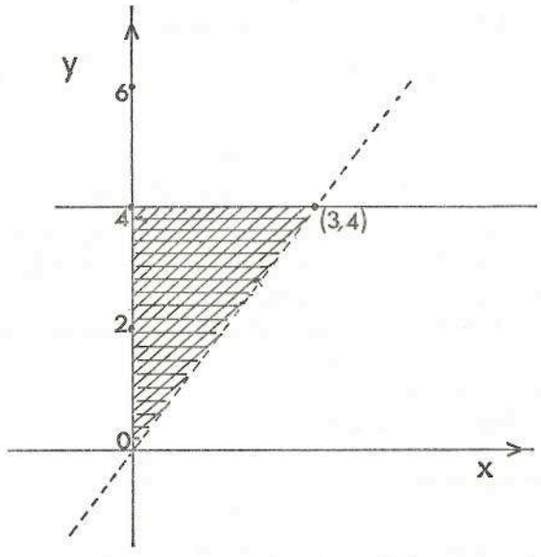
9. (a) Say whether each of the following statements is true or false:

(i) $\left(\frac{1}{2}\right)^2 = \frac{1}{2^2}$

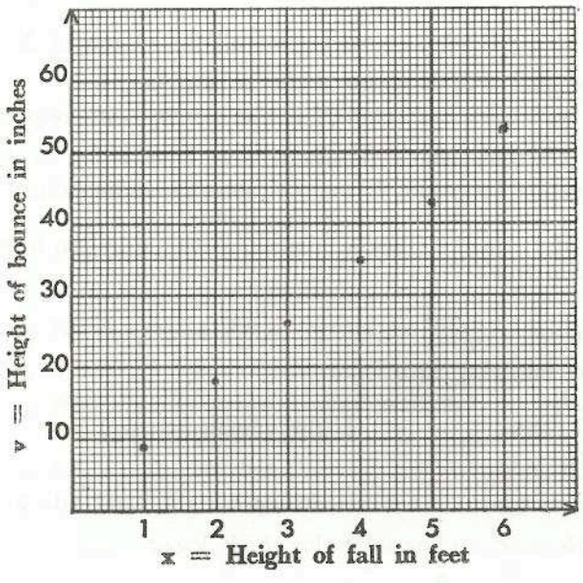
(ii) $a \leqslant a + 1$

(iii) If $x < y$ then $1 - x < 1 - y$.

(b) State the set of inequalities satisfied by the co-ordinates (x, y) of all points in the shaded region of the following diagram and of no other points. (A full line indicates that the boundary is included with the shaded region, and a dotted line that it is excluded.)



10. A golf ball is allowed fall from heights of 1 foot, 2 ft., 3 ft., 4 ft., 5 ft. and 6 ft. After each fall the height of the first bounce of the ball is recorded, and the information is shown in this graph:



Find from the graph as accurately as you can how high does the ball bounce after a fall of 4 feet.

Give an equation involving x and y (see graph) which the graph suggests.

If the ball falls from a height of 6 feet and bounces three times estimate the height of the third bounce as best you can, assuming that the second and further bounces follow the pattern of those shown in the graph.