

MATHEMATICS - HIGHER COURSE - PAPER II

Six questions to be attempted. All questions carry equal marks.

N is the set of natural numbers. Z is the set of integers. R is the set of real numbers.

1. By how much does the compound interest on £875 for 3 years at 4% per annum exceed the simple interest on £875 for the same period at the same rate?

2. Define Relation and Function. Is a function always a relation? Explain your answer.

If $A = \{(1, 2), (4, 3), (2, 3), (3, 7)\}$ and $B = \{(1, 2), (3, 6), (2, 2), (1, 3)\}$, say which relation is a function.

Write down the couples of $A \circ B$ and $B \circ A$ and say whether or not the composite relations are functions.

Calculate $A \circ A \circ A$.

3. (a) If $p, q \in R$ say which, if any, of the following is true:

(i) $p < q \Rightarrow 4 - p < 4 - q$

(ii) $p > q \Rightarrow \frac{1}{q} > \frac{1}{p}$

(iii) $\frac{p}{q} > pq, 0 < q < 1, p < 0$.

(b) Find the least value of $n \in N$ for which

$$1 - (0.99)^n > 0.8.$$

4. A shopkeeper bought an article for £10 and sold it to a customer at a profit. The customer then sold the article for £15 and his profit was 5 per cent greater than the percentage profit made by the shopkeeper. Find how much the customer paid for the article.

5. (a) The first three terms of a sequence are $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}$.

Show that the sequence is neither arithmetic nor geometric.

(b) The n th term of a sequence is $(-1)^{n+1} 2^{n+1}$. Show that the sequence is geometric and find the sum of the first 15 terms.

If S_n is the sum of the first n terms of this sequence and T_n is the n th term, show that

$$3S_n = 2T_n + 4.$$

[Note: $(-1)^{n-1} = (-1)^{n+1}$]

6. (a) Divide the binary number 11,011 by the binary number 11. Check your result by converting to denary notation.

(b) Write down the values of

$$\left(\frac{1}{16}\right)^{\frac{1}{2}}; \log_2 8; \log_8 4.$$

Given that $\log_4 k = \frac{3}{4}$ find $\log_8 k$.

7. Draw the graph of $x^2 - 3x - 5 (=y)$ for real values of x , from $x = -3$ to $x = 5$. Use your graph to find the domain of values of x for which

(i) $x^2 - 3x - 5 \leq 0$,

(ii) $-3 < x^2 - 3x - 5 < 3$.

Use the graph of $y = -3x (x \in R)$ together with above graph to estimate $\sqrt{5}$.

8. (a) Which of the following illustrates that the operation $+$ is commutative:

(i) $3 + 5 = 5 + 3$, (ii) $4 + 3 = 5 + 2$

Give two examples of operations which are non-commutative.

(b) Solve each of the following:

(i) $x = \sqrt{6 + x}$, (ii) $2y = 6 - x$
 $x = y + 1$

Give a geometrical explanation of your answer to (ii).

9. A shipload of fruit arrives at a port during a strike of dock workers and cannot be unloaded.

After one week the fruit begins to rot. Each day $\frac{2}{5}$ of the fruit which was sound the previous day has rotted. If the ship carried 3,000 tons of fruit valued at £180,000,

(i) draw a trend graph to show the decay of the fruit during the ship's second week in port,

(ii) find on which day has half the load rotted,

(iii) find in £ the average daily loss during the ship's second week in port.

10. Graph the lines (i) $y = 2x$, (ii) $y = -2x$.

Indicate the set of points (x, y) which simultaneously satisfy the conditions:

$$y \geq 2x; y \geq -2x; x + y \leq 1.$$

Find a point (x, y) in that set such that $x - y$ is maximum, and write down that maximum.