AN ROINN OIDEACHAIS

INTERMEDIATE CERTIFICATE EXAMINATION, 1969

MATHEMATICS - HIGHER COURSE I

WEDNESDAY, 11th JUNE - Morning 9.30 to 12

SIX questions to be answered

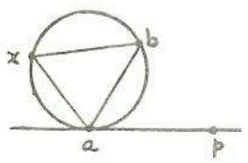
Mathematical Tables may be had from the Superintendent

1. The height of a cone is 8 inches. Its volume is 88 cubic inches. Find the radius of its base to one place of decimals. (Take $\frac{22}{7}$ as an approximation for π)

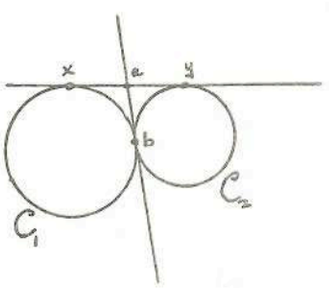
2. A sum of money was to be divided between A, B and C in proportion to the numbers 2, 3, and 4 respectively. In error the money was divided in proportion to the numbers 1, 2, and 3 respectively.

(i) Who lost money because of the error?
(ii) If one person gained £1·50 because of the error, what sum of money was divided?

3. (a) In the diagram ap is tangent to the circle at a, and $[ab]$ is a chord.
Prove $\angle bap = \angle axb$.



(b) Two circles C_1 and C_2 touch at a point b (See diagram). The line ab is a tangent at b to each circle. The line xy is a tangent to circle C_1 at x and to circle C_2 at y. The tangents intersect in a.
Prove that a is the mid-point of the line segment $[xy]$.



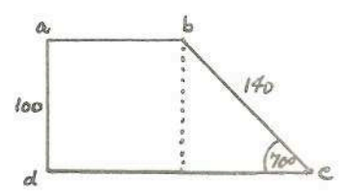
4. (a) Show, without proof, how to construct the bisector of a line segment $[ab]$.
(b) Prove that the perpendiculars at the mid-points of the sides of a triangle are concurrent.

5. (a) The images of the points a and b under reflection in the line $x = 0$ have coordinates (3, 4) and (-2, 3) respectively. Find the coordinates of a and b.
(b) Write down the equation of the line containing $c(1, 2)$ which is
(i) parallel to the line $y = x$.
(ii) perpendicular to the line $y = x$.

6. (a) Find the coordinates of the two points of intersection of the line $y = x - 2$ and the circle $x^2 + y^2 = 4$. Draw a sketch.
Find the distance between these two points.
(b) Prove that the point (1, 2) is outside the circle $x^2 + y^2 = 4$.

7. (a) Use tables to evaluate: $\sin 50°$, $\sin 150°$, $\cos \frac{3\pi}{4}$.
(b) Draw the graph of $\sin x$ $(0 \leqslant x \leqslant 2\pi)$.
Use your graph to find two solutions of the equation $2\sin x = 1$.

8. (a) In a triangle whose angles are $\angle A$, $\angle B$ and $\angle C$
prove $\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$, where a and b are the lengths of the sides opposite $\angle A$ and $\angle B$, respectively.
(b) A field $abcd$ is in the shape of a square combined with a triangle (see diagram).
The sides $[ad]$ and $[bc]$ are 100 yards long and 140 yards long, respectively.
Calculate the area of the field, given $\angle bcd$ measures 70°. (Give your answer to the nearest square yard.)



9. (a) a and b are points in the plane Π. S_a and S_b are central symmetries whose centres are a and b respectively.
(i) Draw a diagram to show that $S_b \circ S_a$ is a translation.
(ii) Is $S_a \circ S_b = S_b \circ S_a$? Explain.
(b) S_x and S_y are central symmetries.
S_y is the inverse of S_x.
What can you say about the point x and the point y?

10. (a) (i) The composite of two reflections in lines which intersect is a rotation. Illustrate by a diagram.
(ii) Draw a diagram to illustrate that the composite of two reflections in lines which are parallel is a translation.
(b) X is a line.
S_X is a reflection in X.
Sketch a diagram of $S_X \circ S_X$.