I N T E R M E D I A T E   C E R T I F I C A T E   E X A M I N A T I O N, 1968

MATHEMATICS - GEOMETRY

MONDAY, 17th JUNE - Morning, 10 to 12.30

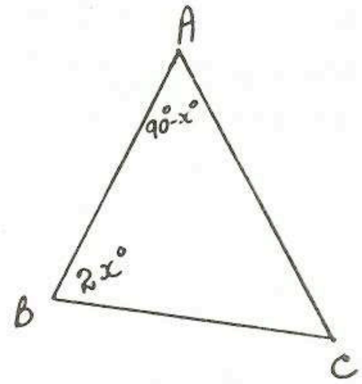
Six questions to be answered.

Mathematical Tables may be had from the Superintendent.

1. If two sides of a triangle are equal, prove that the angles opposite those sides are also equal.

State the converse theorem.

In a triangle ABC there are $2x$ degrees in the angle ABC and $(90 - x)$ degrees in the angle BAC (see diagram). Prove that the triangle ABC is isosceles.



(30 marks)

2. (i) AB is a straight line and S is any point not in AB. Show, using ruler and compass, how to construct a straight line through S parallel to AB. (No proof required).

(ii) Construct a triangle ABC in which AB = 3", $\angle$ACB = 120°, BD = $1\frac{1}{2}$" where D is the foot of the perpendicular from B to AC produced.

(30 marks)

3. In a quadrilateral, if one pair of opposite sides are equal and parallel, prove that the other pair of opposite sides are also equal and parallel.
ABCD is a parallelogram and E is any point on CD (between D and C). DC is produced to F so that CF = DE and AE is produced to M so that EM = AE. Prove that each of the quadrilaterals ABFE and BFME is a parallelogram.

(30 marks)

4. On a given line show how to construct segments of a circle (one on either side of the line) such that each segment contains an angle equal to a given angle.
Hence show how to construct a triangle ABC, given BC, $\angle$BAC and such that A is on a given line. Is it always possible to construct such a triangle ? Draw separate diagrams to illustrate the cases where 0, 1, 2, 3, 4 such triangles can be drawn.

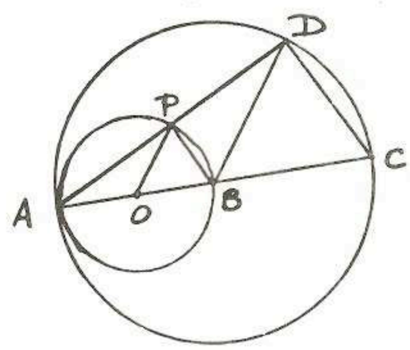
(35 marks)

5. Prove that the perpendicular from the centre of a circle to a chord of the circle bisects the chord.

AB is a radius of the circle ADC. Another circle drawn on AB as diameter has centre O (see diagram). APD is a straight line cutting the circles at P and D as shown in the diagram. Prove that

(i) P is the mid-point of AD.

(ii) $\angle$OPB = $\angle$BDC.



(35 marks)

6. (i) Without using the tables, construct and mark an angle A so that tan A = $1\frac{1}{2}$.

(ii) In a triangle ABC, tan $\angle$BAC = $1\frac{1}{2}$, the perpendicular from B to AC is 2 inches in length and sin $\angle$BCA = $\frac{4}{5}$. Calculate the area of the triangle.

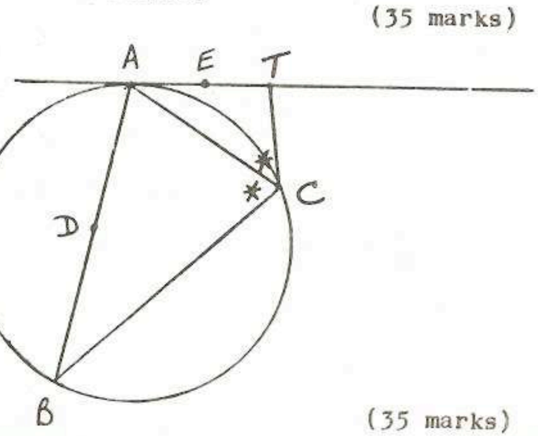
(35 marks)

7. When are two triangles said to be similar?
If two triangles have an angle of one equal to an angle of the other and the sides about the equal angles are proportional, prove that the triangles are similar.
ABC is a triangle inscribed in a circle as shown in diagram. AT is the tangent at A and $\angle$ACB = $\angle$ACT. D and E are the mid-points of AB and of AT, respectively. Prove

(i) Triangles ABC, ATC are equiangular and hence that $\frac{AB}{AT} = \frac{BC}{AC}$.

(ii) Triangles DBC, EAC are similar.

(iii) $\frac{DC}{EC} = \frac{AB}{AT}$.



(35 marks)