

INTERMEDIATE CERTIFICATE EXAMINATION, 1967

MATHEMATICS - GEOMETRY

TUESDAY, 13th JUNE - Morning, 10 to 12.30

Six questions to be answered.

Mathematical Tables may be had from the Superintendent.

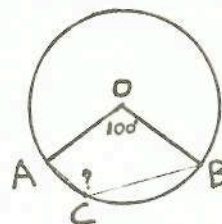
1. Describe, with the aid of a diagram, how to bisect a given angle, and give proof.

If a straight line perpendicular to the bisector of an angle cuts the arms of the angle at p , p_1 , respectively, prove that the bisector bisects the line segment pp_1 .

(30 marks)

2. Prove that an angle at the centre of a circle is double an angle at the circumference standing on the same arc.

A and B are two points on the circumference of a circle of centre O, and C is a point on the minor arc AB as in diagram. If $\angle AOB = 100^\circ$, find, without measurement, how many degrees there are in $\angle ACB$.



(30 marks)

3. Prove that a diagonal of a parallelogram bisects the parallelogram.

Hence prove that the area of a parallelogram is twice the area of any triangle on the same base and between the same parallels.

P is any point inside a given parallelogram ABCD. Prove that the sum of the areas of the triangles PAB, PCD is constant whatever point inside ABCD is taken as P.

(30 marks)

4. Prove that the angles made by a tangent to a circle with a chord drawn from the point of contact are equal, respectively, to the angles in the alternate segments of the circle.

The inscribed circle of a triangle ABC touches the sides at D, E, F. (i) If $\angle ABC = 30^\circ$ and $\angle BCA = 50^\circ$ find, without measurement, how many degrees there are in the angles DEF, EFD, FDE. (ii) In general if the angles of the triangle ABC are α , β , γ prove that the angles of the triangle DEF are $\frac{1}{2}(\alpha + \beta)$, $\frac{1}{2}(\beta + \gamma)$ and $\frac{1}{2}(\gamma + \alpha)$.

(35 marks)

5. (a) Is the following statement true, where AB, CD, EF are straight lines: "If $AB \perp CD$ and $CD \perp EF$, then $AB \perp EF$ "? Draw a diagram to illustrate your answer.

(Note: " $AB \perp CD$ " means "AB is perpendicular to CD")

- (b) A and B are two fixed points 4" apart in a plane. Indicate as clearly as you can, with the aid of four separate diagrams,

- (i) the two points (in the plane) that are 3" from A and 3" from B,
- (ii) all the points (in the plane) that are more than 3" from A and less than 3" from B,
- (iii) all the points (in the plane) that are more than 3" from A and not more than 3" from B,
- (iv) all the points that are included in (iii) but are not included in (ii).

(35 marks)

6. In a triangle ABC the internal bisector of the angle BCA cuts AB at D. Prove $AD : DB = AC : CB$.

The internal bisector of the angle CAB cuts CB at E. Prove that if DE is parallel to AC the triangle ABC is isosceles.

(35 marks)

7. (a) Using the Tables find the values of $\sin 40^\circ$, $\sin 140^\circ$, $\cos 70^\circ$, $\cos 110^\circ$, $\tan 82^\circ$. If a is greater than b , where a and b are numbers, does it follow that $\sin a^\circ$ is greater than $\sin b^\circ$? Give the reason for your answer.

- (b) A searchlight, situated at ground-level at A, directs a narrow beam of light at a point B on the base of a cloud. An observer standing 40 yards from A at ground-level finds that the angle of elevation of B is 82° . Calculate, correct to the nearest 10 feet, the height above ground-level of the base of the cloud

- (i) if the beam of light is vertical,
- (ii) if the beam is inclined towards the observer making an angle of 2° with the vertical.

(35 marks)