INTERMEDIATE CERTIFICATE EXAMINATION, 1966

MATHEMATICS (Algebra)

FRIDAY, 10th JUNE — Afternoon, 2.30 to 5

ALL questions to be answered.

Mathematical Tables may be obtained from the Superintendent.

1. (i) Solve the equation

$$\frac{2x - 1}{3} = 1 - \frac{1 - x}{2}.$$

(ii) Write down any three values of x for which $5 - 3x$ is positive and any three values
for which $5 - 3x$ is negative.

(28 marks)

2. (a) Factorise:

(i) $ax + b^2 - bx - ab$;

(ii) $(x + 1)^2 + 2(x + 1) - 15.$

(b) When $x^3 + px + q$ is divided by $x - 1$, the remainder is $- 12$ and when divided by $x - 2$,
the remainder is also $- 12$. Find the value of p and the value of q.

(28 marks)

3. Solve the following equations, giving your solutions correct to two significant figures:

(i) $3x^2 - 5x + 1 = 0$;

(ii) $3(2x - 1)^2 - 5(2x - 1) + 1 = 0.$

(28 marks)

4. To prepare for a party a student spent £1 8s. in buying cakes at 2s. 6d. each and buns
at 3d. each. The cost of the cakes and buns that were eaten amounted to £1 2s. and it was
found that $\frac{1}{5}$ of the cakes and $\frac{1}{3}$ of the buns remained. How many of each kind did the student
buy ?

(28 marks)

5. (i) When $x = 1 - \sqrt{2}$, prove that $x^2 - 2x - 1 = 0$ and hence, or otherwise, express in
simplest form the value of $x^3 - 2x^2 - 1.$

(ii) If $4x = (x - a)^2 - (x - b)^2$ for all values of x, find the value of a and the value of
b and hence, or otherwise, express 1360 as the difference of two squares.

(28 marks)

6. (i) If m, n are positive whole numbers, prove $a^m \cdot a^n = a^{m+n}.$

(ii) Solve, without using the tables, each of the following equations:

$8^x = 8$; $\log_{10} x = 3$; $4^x = 8$; $\log_x 9 = 2.$

(iii) If $\log_3 \left(\dfrac{10}{a}\right) = q$ and $\log_{3a} 9 = p$, show that $q = \dfrac{1 - \log_{10} a}{\log_{10} 3}$ and express pq in terms
of $\log_{10} a$.

(30 marks)

7. Draw the graph of $15x - 3x^2$ for values of x from $x = 0$ to $x = 5.$

Find from your graph, as accurately as you can, the range of values of x for which
$15x - 3x^2$ decreases as x increases.

ABC is an equilateral triangle of side 5 inches. A point P is taken on AB, PQ is drawn
parallel to BC cutting AC at Q and QR is drawn parallel to AB cutting BC at R. If PB =
x inches, show that the area of PQRB can be expressed in the form $k(15x - 3x^2)$, where k is
independent of x, and with the aid of your graph find the greatest area of PQRB.

(30 marks)