

INTERMEDIATE CERTIFICATE EXAMINATION, 1965

MATHEMATICS - GEOMETRY

MONDAY, 21st JUNE - Morning, 10 to 12.30

The total number of questions answered should not exceed six.

Mathematical Tables may be obtained from the Superintendent.

1. If two sides of a triangle are equal, prove that the angles opposite those sides are equal.

State the converse theorem.

In a triangle PQR,  $PQ = PR$  and  $\angle PQR = 2\angle QPR$ . If the bisector of  $\angle PRQ$  meets PQ at T, prove that  $PT = QR$ .

(30 marks)

2. Prove that parallelograms on the same base and between the same parallels are equal in area.

Using ruler and compass only, construct

(i) a parallelogram ABCD in which  $AB = AC = 4$  cm. and  $\angle BAC = 45^\circ$ ,

(ii) a square equal in area to the parallelogram ABCD.

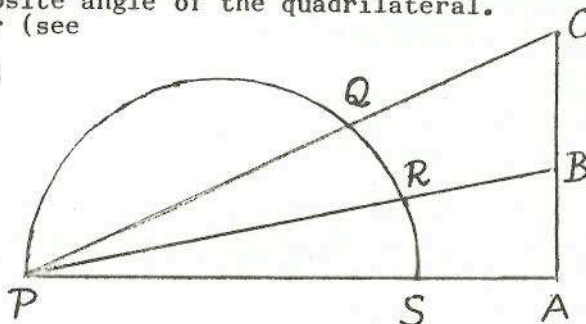
(No proof required in (i) or (ii) but construction lines should be clearly shown.)

(30 marks)

3. Prove that the sum of the opposite angles of a cyclic quadrilateral is equal to two right angles.

Hence, deduce that if one side of a cyclic quadrilateral is produced, the exterior angle so formed is equal to the interior opposite angle of the quadrilateral.

PQRS is a semi-circle on PS as diameter (see diagram). From a point C on PQ produced a perpendicular to PS is drawn so as to meet PS externally at A and PR externally at B. Show that



(30 marks)

4. Draw geometrical diagrams to illustrate the following identities and give a short explanation of your diagrams showing how they illustrate the identities:-

(i)  $(a + b)c = ac + bc$ ,

(ii)  $(a + b)^2 = a^2 + b^2 + 2ab$ .

E is a point on the side AB of the square ABCD. Prove that

$$DE^2 = 2AE \cdot AB + BE^2.$$

(35 marks)

5. (i) Show how to construct on a given line a segment of a circle which contains an angle equal to a given angle.

(ii) ABC is a triangle in which AB is greater than AC. P is a point in AB such that  $AP = AC$ . Prove that  $\angle BPC = 90^\circ + \frac{1}{2}\angle BAC$ .

Using (i) and (ii), or otherwise, show how to construct a triangle ABC given that  $BC = 6$  cm.,  $\angle BAC = 60^\circ$  and the difference between AB and AC is 4 cm.

(35 marks)

6. If two triangles have one angle of one equal to one angle of the other and the sides about the equal angles in proportion, prove that the triangles are similar.

PQR is a triangle in which  $PQ = 2PR$ . The bisector of the angle QPR meets QR in S and T is the midpoint of PS. Prove that the triangles PQS and PRT are similar and deduce that  $RT = RS$ .

(35 marks)

7. (a) Using ruler and compass only, construct the angle A such that  $\tan A = 1.5$ .

(b) A ship leaves a certain port at 3 p.m. and travels at 15 m.p.h. in a direction  $50^\circ 38'$  south of west. At 7 p.m. the ship alters course and travels due west at the same speed.

From a second port, 40 miles due east of the first one, a speedboat sets out at 4 p.m. and sailing on a straight course intercepts the ship at 9 p.m.

Calculate, to the nearest minute, the direction in which the speedboat travels and also its speed to the nearest mile per hour.

(35 marks)