AN ROINN OIDEACHAIS

INTERMEDIATE CERTIFICATE EXAMINATION, 1965

MATHEMATICS (Algebra)

WEDNESDAY, 23rd JUNE – MORNING, 10 to 12.30.

ALL questions to be answered.
Mathematical Tables may be obtained from the Superintendent.

1. (i) Solve the equation:

$$\tfrac{2}{3}(3x - 2) - \tfrac{3}{2}(2x - 3) = 9\tfrac{1}{6}.$$

(ii) Solve the simultaneous equations

$$2x - 9y = 4,$$
$$6x - 6y = 5.$$

(28 marks)

2. (i) Show that $(x + 2)$ is a factor of $x^3 + 8x^2 - 79x - 182$ and find the remaining factors.

(ii) Solve the equation $x^2 = 16x + 561$.

(28 marks)

3. Factorise fully:

(i) $(a - b + c)x - a + b - c$,

(ii) $36 + 7p^2 - 4p^4$,

(iii) $a(3a + 2) - b(3b - 2)$.

If $t^2 + 2t = 1$, prove $t^4 - 6t^2 + 1 = 0$.

(28 marks)

4. 252 students take part in a drill display. 240 arrive early and are arranged in rows with an equal number of students in each row. Two students are then taken from each row and these together with the 12 students who arrive later form three further rows. Every row of the display now contains the same number of students. Into how many rows are the 252 students arranged and how many students are then in each row ?

(28 marks)

5. (i) Find the value of x for which $\sqrt{2x + 3}$ exceeds $\sqrt{3x - 8}$ by 2.

(ii) If a and b are both positive and b is greater than a, show that $\frac{1}{2}(a + b)$ is greater than a and less than b.

(28 marks)

6. (i) Without using the Tables, write down the values of

$$3^3,\ (4)^{1\frac{1}{2}},\ (\tfrac{1}{4})^{-2},\ \log_4 8.$$

(ii) Prove that $\log_b a = \log_c a \div \log_c b$.

(iii) If $\log_8 9 = p$ and $\log_3 5 = q$, show that $p = \dfrac{2\log_{10} 3}{3\log_{10} 2}$ and express $\log_{10} 2$ in terms of p and q.

(30 marks)

7. Draw the graph of $\frac{x^2}{10} + \frac{10}{x}$ for values of x from $x = 1$ to $x = 6$.

Find from your graph, as accurately as you can, the values of x for which $\frac{x^2}{10} + \frac{10}{x} = 5$.

The cost, in shillings per hour, of running a boat at a speed of x m. p. h. is given by $\frac{x^3}{100} + 1$. Use your graph to find the speed at which the boat should travel a distance of 10 miles so as to keep the cost as low as possible.

(30 marks)