AN ROINN OIDEACHAIS

INTERMEDIATE CERTIFICATE EXAMINATION, 1964.

MATHEMATICS — GEOMETRY.

FRIDAY, 5th JUNE. — Morning, 10 to 12.30.

The total number of questions answered should not exceed six.

Mathematical Tables may be obtained from the Superintendent.

1. If two sides of a triangle are equal, prove that the angles opposite these sides are equal.

In a triangle ABC, AB = AC. The side BC is produced to P such that AC = CP. Show that $\angle ABC = 2\angle APB$.

If PA = PB, find the size of $\angle APB$.

(30 marks.)

2. (i) AB is a straight line and S is any point not in AB. Show how to draw a straight line through S parallel to AB. (No proof required.)

(ii) Show how to construct a triangle equal in area to a given quadrilateral. Give proof.

(30 marks.)

3. Prove that the straight line joining the middle points of two sides of a triangle is parallel to the third side.

S is any point on the side PQ of a given triangle PQR and T is the mid-point of RS. Find the locus of T as S moves along PQ. Give proof.

(30 marks.)

4. Two chords of a circle, AB and CD, cut one another internally at P. Prove that AP.PB = CP.PD.

A circle which is concentric with the circle ABCD passes through P and cuts AB at Q. CQ produced meets the circle ABCD at E. Prove (i) AP = QB, (ii) CP.PD = CQ.QE.

(35 marks.)

5. In an obtuse-angled triangle prove that the square on the side opposite the obtuse angle is greater than the sum of the squares on the other two sides by twice the rectangle contained by one of those sides and the projection of the other side upon it.

In a triangle ABC, AB = 8 cm., BC = 5 cm., CA = 6 cm. Show that the $\angle ACB$ is obtuse and find the length of the projection of AC on BC.

(35 marks.)

6. Prove that if two triangles are equiangular, their corresponding sides are proportional.

ABC and DBC are two triangles on the same base BC and on the same side of BC. AB is parallel to CD and AC meets BD at P. A line through P parallel to AB meets BC at Q. Name a triangle that is similar to the triangle ABC and a triangle that is similar to the triangle DBC. Hence, show that AB.QC = DC.BQ.

(35 marks.)

7. (i) There are 40° in an angle AOB. If AO = 7 cm., find the length of the shortest distance from A to OB.

(ii) In a triangle ABC, $\angle BAC = 50^\circ$, $\angle ACB = 90^\circ$. D is a point on BC such that $\angle BAD = 25^\circ$ and BD = 5 cm. Calculate (i) the length of DC, (ii) the area of the triangle ABC. Give your answer in each case correct to two significant figures.

(35 marks.)