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(Department of Education.)

BRAINNSE AN MHEADHON-OIDEACHAIS
(Secondary Education Branch).

INTERMEDIATE CERTIFICATE EXAMINATION, 1939.

MATHEMATICS (Geometry).

THURSDAY, 15*th JUNE*.—MORNING, 10 A.M. TO 12.30 P.M.

The total number of questions answered should not exceed *six*.

Mathematical Tables may be obtained from the Superintendent.

Candidates should state the text-book used in order to indicate the sequence followed.

1. The side BC of a triangle ABC is produced to X: prove that

(i) the angle XCA is greater than the angle BAC;

(ii) the sum of BA and BC is greater than twice the line joining B to the mid-point of AC.

[30 marks.]

2. PQR is a triangle in which PQ=PR; the side QP is produced to S so that PS=QP. Prove that SR is perpendicular to QR.

Show how the above may be used to draw a perpendicular to a line XY at the point Y.

[30 marks.]

3. ABCD is a parallelogram. Through a point P on the diagonal AC a line EPF is drawn parallel to AB, and a line GPH is drawn parallel to BC: these lines meet BC, CD, DA, AB at F, H, E, G respectively. Prove that PGBF is equal in area to PHDE.

[30 marks.]

4. Prove that the sum of the opposite angles of a cyclic quadrilateral is equal to two right angles.

Two circles intersect at A and B. Through A and B lines PAQ, RBS are drawn meeting one of the circles again at P, R respectively and meeting the other again at Q, S respectively. Prove that PR is parallel to QS.

[30 marks.]

5. Show with proof how to draw a circle which shall pass through the three vertices of a triangle.

The centre of the circumcircle of a triangle may lie

(i) within the triangle, *or*

(ii) on a side of the triangle, *or*

(iii) outside the triangle.

State for what kinds of triangles those statements (i), (ii), (iii) are respectively true.

[30 marks.]

6. Prove that the square on the hypotenuse of a right-angled triangle is equal in area to the sum of the squares on the other two sides.

Construct two squares, P and Q, such that Q=2P.

[30 marks.]

7. Illustrate the following identity by a geometrical diagram:

$$(a+b)^2-(a-b)^2\equiv 4ab.$$

The diagram should be clearly marked and a brief explanation given showing how it satisfies the identity.

[35 marks.]

8. AB and CD are two lines: show how to find a point P in AB such that $AP.PB=CD^2$.

Prove that P cannot lie between A and B if 2CD is greater than AB.

[35 marks.]

9. ABC is a triangle. The sides BC, CA, AB are produced to D, E, F respectively so that BD=2BC, CE=2CA, AF=2AB. The points D, E F are joined. Prove that

(i) $DE^2=2CA^2+2AD^2-BC^2$,

(ii) $DE^2=6CA^2+3BC^2-2AB^2$,

(iii) $DE^2+EF^2+FD^2=7(BC^2+CA^2+AB^2)$.

[35 marks.]

10. A post, whose height is a feet, and a chimney stand on the same horizontal plane. Viewed from the foot of the post the angle of elevation of the top of the chimney is x° and viewed from the top of the post the angle of elevation of the top of the chimney is y°. Express in terms of a, x, y,

(i) the height of the chimney,

(ii) the distance between the foot of the post and the foot of the chimney.

Calculate the values of those expressions when $a=25, x=42\frac{4}{5}, y=17\frac{1}{2}$.

[35 marks.]