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INTERMEDIATE CERTIFICATE EXAMINATION, 1937.

MATHEMATICS (Geometry).

THURSDAY, 17*th* *JUNE*.—Morning, 10 a.m. to 12.30 p.m.

The total number of questions answered should not exceed *six*.

Mathematical Tables may be obtained from the Superintendent.

Candidates should state the text-book used in order to indicate the sequence followed.

1. Define a "*parallelogram*" and prove that the opposite sides of a parallelogram are equal.

ABCD is a parallelogram in which the diagonal AC bisects the angle BAD. Prove that ABCD is equilateral.

[30 marks.]

2. Prove that the sum of two opposite angles of a cyclic quadrilateral is equal to two right angles.

State and prove the converse theorem.

[30 marks.]

3. Prove that parallelograms on the same base and of equal altitudes are equal in area.

ABCD is a quadrilateral whose diagonals are AC, BD. The parallels to BD drawn through A and C meet the parallels to AC drawn through B and D at P, Q, R, S respectively: prove that PQRS=2ABCD.

[30 marks.]

4. Show, with proof, how to inscribe a circle in a triangle.

The angles A, B, C of a triangle ABC are in the ratio 2 : 3 : 4 and the inscribed circle touches BC, CA, AB at P, Q, R respectively. Calculate the number of degrees in each of the angles of the triangle PQR.

[30 marks.]

5. Draw geometrical diagrams to illustrate the identities:

(i) $a^2-b^2=(a+b)(a-b)$,

(ii) $(a-b)^2=a^2-2ab+b^2$

when a and b are positive numbers and $a>b$.

[The diagrams should be marked clearly and a very brief description given showing how they illustrate the identities.]

[30 marks.]

6. AC is a diagonal of a square ABCD. On the side of AC remote from D an equilateral triangle ACE is described. Through B lines are drawn parallel to EA, EC and meeting AD, DC at G, F respectively. Prove that the triangle BGF is equilateral.

[30 marks.]

7. Two chords of a circle, PQ, RS intersect at X: prove that PX.XQ=RX.XS.

Hence using a large scaled diagram drawn accurately on squared paper evaluate $(2{\cdot}3\times1{\cdot}9)\div1{\cdot}6$.

[35 marks.]

8. From a point O outside a circle two lines are drawn, one intersecting the circle at A, B and the other touching it at T: prove that $OT^2=OA.OB$.

Show how to describe another circle which shall pass through A and B and touch another line OX passing through O.

[35 marks.]

9. ABCD is a rectangular sheet of paper in which BC=4 ins. P is a point on CD such that CP=3 ins. The paper is folded so that the corner B coincides with P: calculate the length of the crease.

[35 marks.]

10. Prove the relation $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}$, where ABC is any triangle.

From a point P on top of a cliff x feet above sea-level two fishing boats, A, B are seen in a line with the foot of the cliff. The angles of depression of A and B are α and β respectively ($\alpha>\beta$). Find in simplest form in terms of x, α, β expressions for (i) the distance of A from the foot of the cliff; (ii) the distance between A and B.

Simplify the results when $x=750$, $\alpha=21°$, $\beta=14° \, 40'$.

[35 marks.]