

APPLIED MATHEMATICS - ORDINARY LEVEL

WEDNESDAY 23 JUNE - AFTERNOON 2.00 TO 4.30

Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

Take the value of g to be 10 m/s^2 .

\vec{i} and \vec{j} are unit perpendicular vectors in the horizontal and vertical directions, respectively.

**Marks may be lost if all your work is not clearly shown
or you do not indicate where a calculator has been used.**

1. A car A, accelerating uniformly, passes a point p with a speed of 5 m/s . 15 seconds later it reaches point q and passes q with a speed of 20 m/s .
Draw an accurate speed-time graph for the motion of car A between p and q .

Find A's acceleration and find, also, $|pq|$.

At the same time as A passes p , a second car B also passes p but with an initial speed of 6 m/s and accelerates uniformly to arrive at q at the same time as A.

What is B's acceleration?

Find, also, B's velocity at the point q .

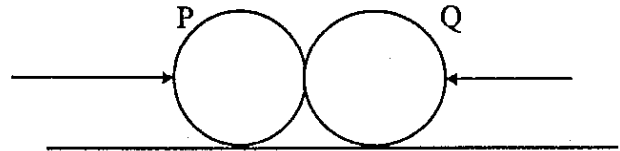
2. A ship D travelling with a velocity of $3\vec{i} - 4\vec{j} \text{ m/s}$ is 5 km due west of ship C. Ship D sends out a distress signal which is picked up by C and C travels immediately with a velocity of $2\vec{i} - 4\vec{j} \text{ m/s}$ so as to come to the assistance of D.

- (i) What is D's speed and direction?
(ii) Find the magnitude and direction of the velocity of C relative to D.
(iii) How long, to the nearest minute, will it take C to reach D?

3. A projectile is fired upwards from the top of a cliff with a velocity of 25 m/s at an angle A to the horizontal. $\tan A = \frac{3}{4}$.

The projectile strikes the sea at a distance d from the foot of the cliff. Calculate d .

4. Two smooth spheres P and Q collide directly on a smooth horizontal table. The mass of P is twice that of Q. Before collision, P and Q are moving in opposite directions with equal speeds. As a result of the collision sphere P comes to rest.

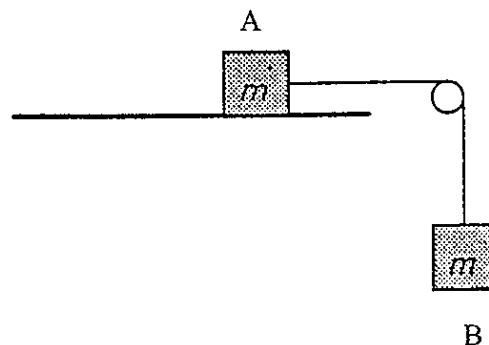


Find the coefficient of restitution for the collision.

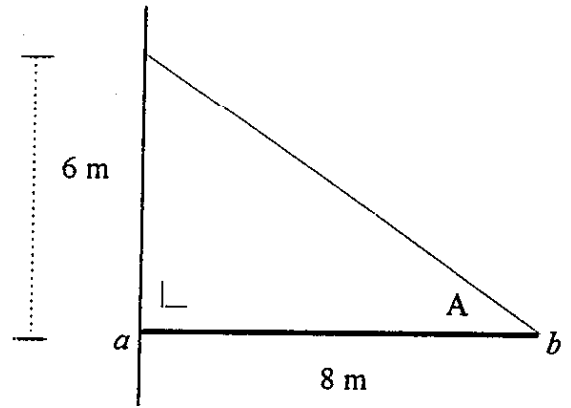
Find, also, the fraction of the total kinetic energy lost as a result of the collision.

5. Two blocks, A and B, each of mass m , are connected by a light inextensible string passing over a smooth pulley at the edge of a smooth table. B hangs freely under gravity.

- (i) When the system is released from rest, find the common acceleration of the blocks.
- (ii) If the smooth table is replaced by a rough table and the system is released from rest, as before, the common acceleration of the blocks is half what it was in the first case. Find this coefficient of friction between A and the rough table.



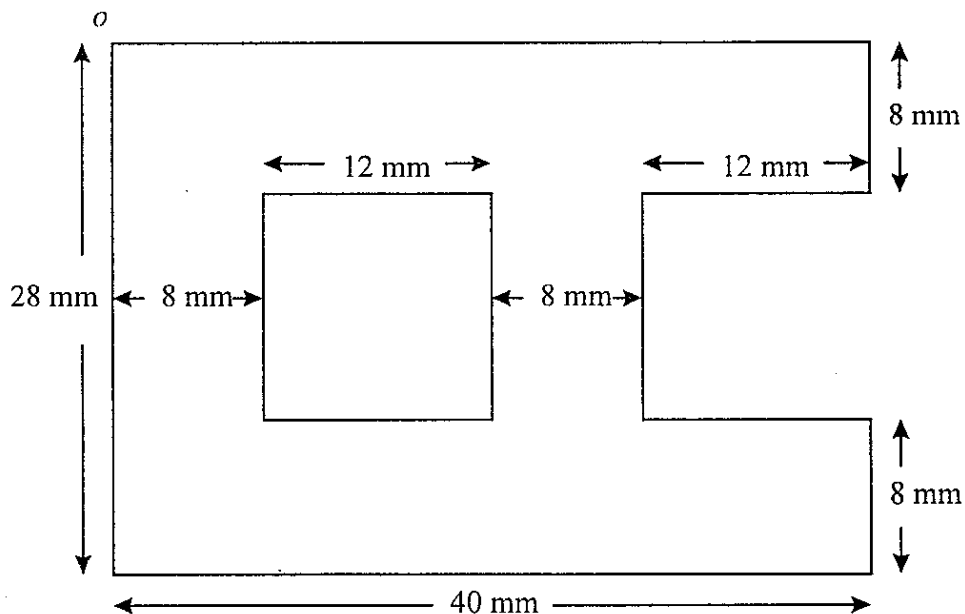
6. The diagram shows a uniform beam $[ab]$, which is 8 m long and has a mass of 20 kg. The beam is freely hinged at a to a vertical wall and held in a horizontal position by a light, inextensible string attached to b and to a point on the wall 6 m vertically above a . The angle between the string and the beam is A , as shown.



Show in a diagram, all the forces acting on the beam.

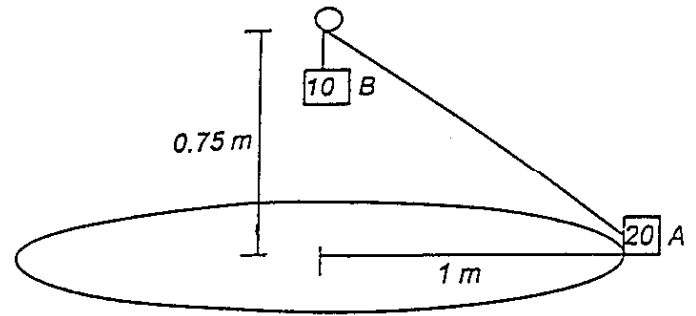
- (i) Find $\sin A$ and $\cos A$.
- (ii) Find the vertical and the horizontal components of the reaction at the hinge.
- (iii) Find the value of the tension in the string.

7. A uniform rectangular piece of metal measures 28 mm \times 40 mm. Two square pieces each of side 12 mm are removed, as in the diagram:



Calculate the distance from the point o to the centre of gravity of the remaining piece, correct to one place of decimals.

8. The diagram shows two particles A and B of mass 20 kg and 10 kg respectively. Particle A is on a smooth horizontal table and is connected to particle B by a light inextensible string passing through a smooth ring fixed at a distance of 0.75 m above the table, so that B hangs freely under gravity and remains at rest. Particle A describes a horizontal circle of radius 1 m, whose centre is vertically below the ring, with constant angular velocity ω radian/s.



Show, on separate diagrams, all the forces acting on A and B.

- Find
- (i) the tension in the string
 - (ii) the angular velocity
 - (iii) the normal reaction between A and the table.

9. A solid sphere, of diameter 21 cm and of density ρ kg/m³, floats in water with half of its volume submerged.

Draw a diagram showing all the forces acting the sphere and find the value of ρ .

The same sphere is then pulled under the surface and is held, totally submerged, by a light inextensible string attached to the sphere and to the bottom of the tank.

Show, in a diagram, all the forces acting on the sphere in this case, and find the tension in the string.

[Density of water = 1000 kg/m³]

[volume of sphere = $\frac{4}{3} \pi r^3$]

[Take $\pi = \frac{22}{7}$]

