AN ROINN OIDEACHAIS

M. 31

LEAVING CERTIFICATE EXAMINATION, 1995

APPLIED MATHEMATICS - ORDINARY LEVEL

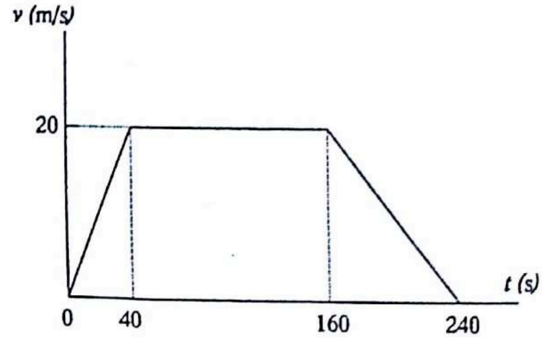
FRIDAY, 23 JUNE – MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 10 m/s^2.
$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively.

Marks may be lost if all your work is not clearly shown or you do not indicate where a calculator has been used.

1. *(a)* A train starts from rest at station P and accelerates uniformly to its maximum speed. It maintains its maximum speed for a period and then decelerates uniformly to come to rest at station Q.

The velocity-time graph shown illustrates the motion of the train from P to Q, the total journey taking 240 seconds.

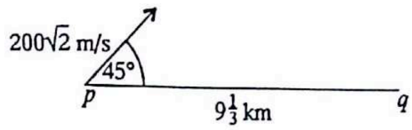


Use the graph to find

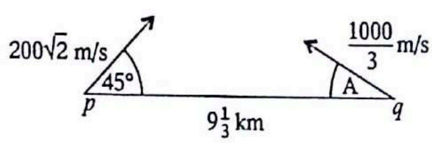
(i) the acceleration of the train
(ii) the deceleration of the train
(iii) the distance from P to Q.

Another train starts from rest at P and accelerates with the same uniform acceleration as in *(i)* for t_1 seconds until it reaches its maximum speed of $20\sqrt{3}$ m/s. On reaching its maximum speed, this train at once decelerates with the same uniform deceleration as in *(ii)* and after t_2 seconds comes to rest at Q.
Find the time taken for this train to travel from P to Q, correct to one place of decimals.

2. A missile is fired from a point p on level horizontal ground with an initial speed of $200\sqrt{2}$ m/s inclined at an angle of 45° to the horizontal, towards q, where $|pq| = 9\frac{1}{3}$ km. How far short of q will the missile hit the ground ?



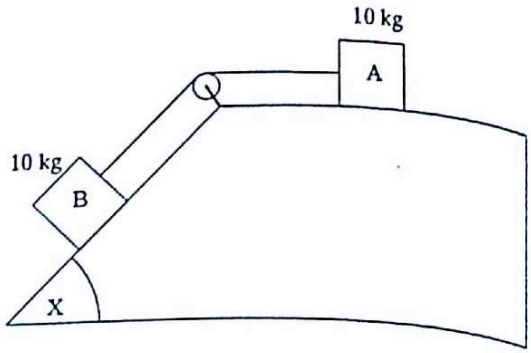
However, at the same instant as the missile is fired from p, a missile is fired from q with an initial speed of $\frac{1000}{3}$ m/s inclined at an angle A to the horizontal, where $\tan A = \frac{3}{4}$, to intercept the missile from p.



Calculate the height above the ground at which the interception (collision) occurs.

OVER→

3. Particle A on a rough horizontal plane is connected by a light inelastic string passing over a smooth pulley at an edge of the plane to particle B on a rough plane inclined at an angle X to the horizontal, where $\tan X = \frac{3}{4}$. The mass of A is 10 kg and of B is 10 kg. The coefficient of friction on each plane is μ. When the system is released from rest, B moves down the inclined plane. The common acceleration of the two particles is 1 m/s^2.



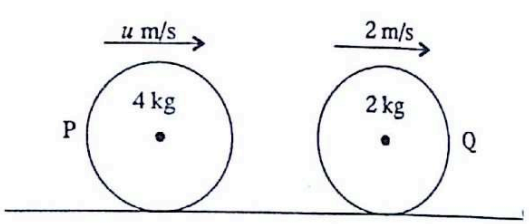
(i) Show, on separate diagrams, the forces acting on each particle.
(ii) Find the value of μ.
(iii) Find the tension in the string.

4. *(a)* The speed of particle A is 4 m/s and the speed of particle B is 3 m/s. Find the velocity of A relative to B when

(i) both A and B are travelling due East
(ii) A is travelling due East and B is travelling due West
(iii) A is travelling due East and B is travelling due North.

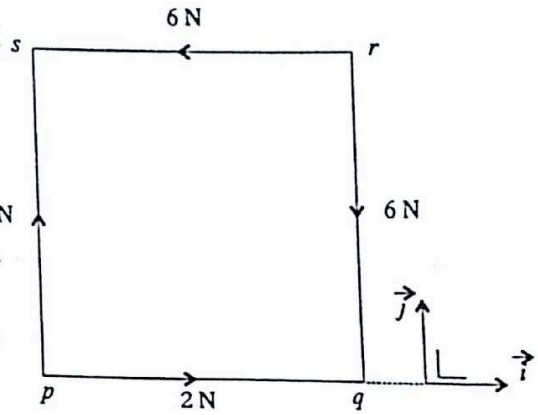
(b) City C is 225 km due North of City D. The scheduled flight time of a plane from D to C is 45 minutes in still air.
If, on a particular day, a strong wind is blowing from the West at 51 km/hr, determine the direction and the speed with which the plane should head in order to complete the journey from D to C in the scheduled time.

5. A smooth sphere P of mass 4 kg moving with a velocity of u m/s collides directly with a second smooth sphere Q of mass 2 kg moving in the same direction with a velocity of 2 m/s. After the collision, P and Q keep moving in the same direction with velocities of v m/s and 3 m/s, respectively. The coefficient of restitution for the collision is ½.



(i) Find the value of u.
(ii) Find the impulse imparted to Q.
(iii) Find the loss in kinetic energy due to the collision.

6. *(a)* *pqrs* is a square with side of length 20 cm. Forces act along the sides of the square as shown.

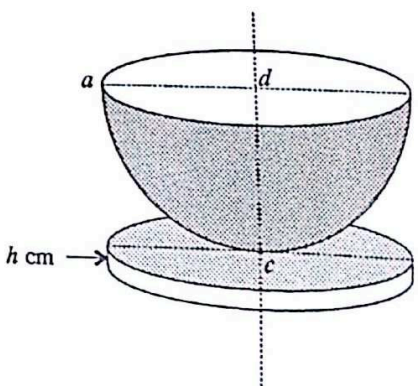


(i) Write down, in terms of $\vec{i}$ and $\vec{j}$, the resultant of the four forces.

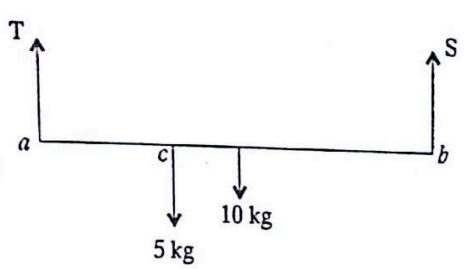
(ii) Find the magnitude of the resultant force.

(iii) Find the distance of the line of action of the resultant from *p*.

(b) An ornamental bowl consists of a uniform cylindrical base of height *h* cm and weight 20W, surmounted by a hemispherical shell with radius of length 20 cm and weight W. The radius of the cylindrical base is also of length 20 cm. The shell and the base are rigidly joined at *c* and the vertical line *cd* is an axis of symmetry of the bowl. If the centre of gravity of the bowl is at *c*, find the value of *h*.
See Tables p. 40.



7. A metre stick, [*ab*], of mass 10 kg supports a mass of 5 kg suspended from *c*. The metre stick is held in a horizontal position by two vertical and inelastic strings, one fixed at each end. The tension in the string at *a* is T and at *b* is S and



$$T : S = 8 : 7$$

(i) Find the value of T and the value of S.

(ii) Find |*ac*|.

If the 5 kg mass were instead to be suspended from *a*, find the ratio

$$T_1 : S_1,$$

where T_1 and S_1 are the new tensions in the strings at *a* and *b*, respectively.

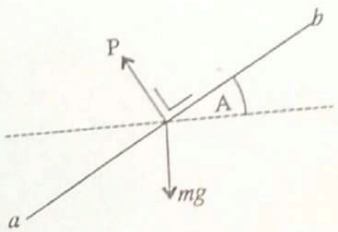
OVER→

8. *(a)* A car, of mass M kg, is rounding a bend on a level road.
The bend forms an arc of a circle of radius 25 m.
The coefficient of friction between the car's tyres and the road is μ.

(i) Show, in a diagram, the three forces acting on the car.

(ii) If the maximum speed with which the car can safely round the bend is $10\sqrt{2}$ m/s, find the value of μ.

(b) An aircraft, of mass m tonnes, flies in a circular path of radius 5⅓ km at a speed of v m/s, waiting for clearance to land.
In order to keep the aircraft in the circular horizontal path, the pilot must bank the aircraft so that the wings (represented by [*ab*] in the diagram) are inclined at an angle A to the horizontal, where $\sin A = \frac{3}{5}$.
The engines supply a lifting force P which acts at right angles to the wings as shown.
Calculate the value of v, the speed of the plane.



9. *(a)* A cylindrical tank with internal radius of length 1.75 m contains oil, of relative density 0.85, to a depth of 4 m.
Calculate the thrust on the horizontal base of the tank due to the oil.

A solid body of mass 750 kg and relative density 2.5 is lowered into the oil by means of a fine wire until it hangs fully immersed in the oil.
Calculate the new depth the oil rises to in the tank, correct to three decimal places.
Take $\pi = \frac{22}{7}$.
Density of water = 1000 kg/m^3.

(b) An hydrometer of mass 0.06 kg just floats vertically in a liquid of relative density 1.2 as shown.

Calculate the volume of the hydrometer.

Density of water = 1000 kg/m^3.