AN ROINN OIDEACHAIS M. 31

LEAVING CERTIFICATE EXAMINATION, 1994 1027

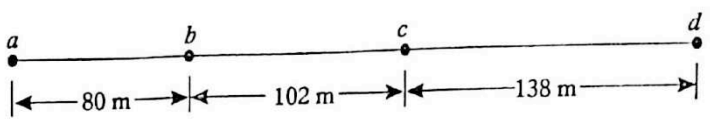
APPLIED MATHEMATICS - ORDINARY LEVEL

FRIDAY, 24 JUNE – MORNING, 9·30 to 12·00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 10 m/s^2.
$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively.

Marks may be lost if all your work is not clearly shown or you do not indicate where a calculator has been used.

1. Four points, a, b, c, and d, are in a straight line such that $|ab| = 80$ m, $|bc| = 102$ m and $|cd| = 138$ m.



A particle moving in a straight line with constant acceleration, f m/s^2, passes the four points taking 8 s to go from a to b and 6 s to go from b to c.

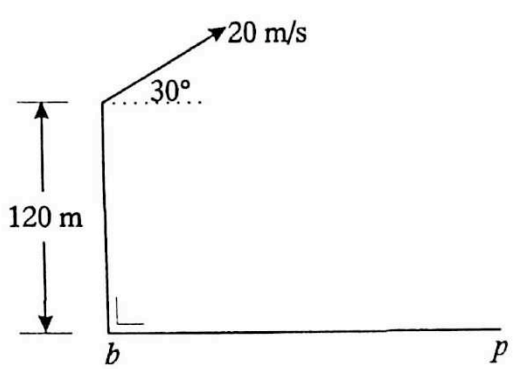
The speed of the particle as it passes the point a is 6 m/s.

(i) Find the value of f, the acceleration.

(ii) Find the speed of the particle as it passes the point c.

(iii) Find the time taken for the particle to travel from a to d.

2. A particle is projected from the top of a cliff, 120 m high, with initial velocity of 20 m/s inclined at an angle of 30° to the horizontal. The particle strikes the sea at the point p.



Find

(i) the greatest height reached by the particle above sea level

(ii) the time it takes the particle to reach p

(iii) $|bp|$, the distance from b, the base of the cliff, to the point p.

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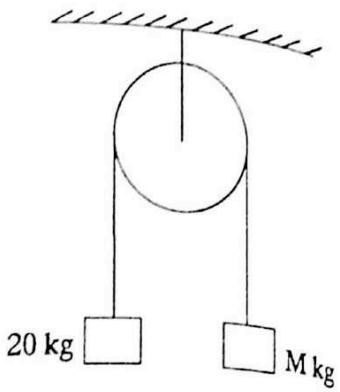
3. Two particles of masses M kg and 20 kg are connected by a light inextensible string passing over a smooth fixed pulley. The particles are released from rest.



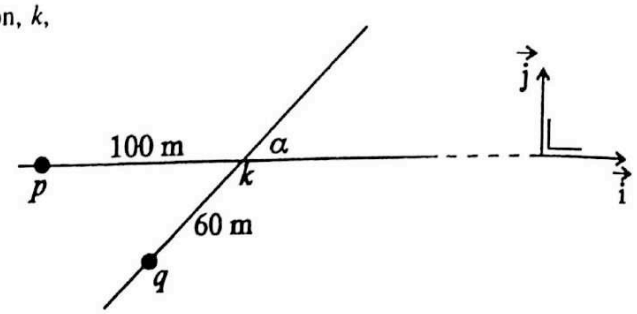
(i) Show on separate diagrams the forces acting on each particle.

(ii) Write down the equation of motion for each particle.

(iii) Find, in terms of M, the acceleration of the particles and the tension in the string.

4. Two straight roads intersect at a junction, k, and at an angle α where $\tan \alpha = \frac{4}{3}$.

At a certain instant the particle p is 100 m from k and is travelling towards k with a velocity of 8 m/s. At the same instant on the other road the particle q is 60 m from k and is travelling towards k with a velocity of 4.8 m/s.



Find, in terms of $\vec{i}$ and $\vec{j}$,

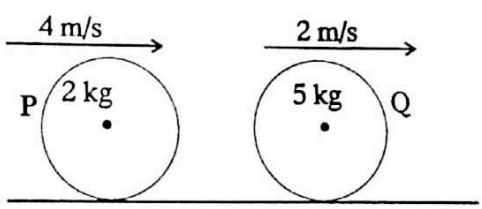
(i) the velocity of p and the velocity of q

(ii) the velocity of q relative to p.

Calculate the shortest distance between p and q.

5. Define Linear Momentum.

A smooth sphere P of mass 2 kg moving with a velocity of 4 m/s collides directly with a second smooth sphere Q of mass 5 kg moving in the same direction with a velocity of 2 m/s. After the collision, P and Q keep moving in the same direction with velocities of x m/s and y m/s, respectively.



If the impulse (change of momentum) imparted to sphere Q as a result of the collision is 5 N, find the value of y.
Hence, find

(i) the value of x

(ii) the value of e, the coefficient of restitution.

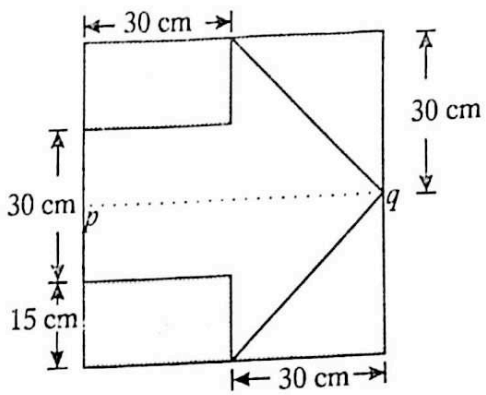
6. *(a)* Four particles are placed on a horizontal plane as follows:

10 kg mass with position vector $-2\vec{i} + 3\vec{j}$
4 kg mass with position vector $-\vec{i} - 2\vec{j}$
7 kg mass with position vector $-\vec{j}$
3 kg mass with position vector $x\vec{i} + y\vec{j}$.

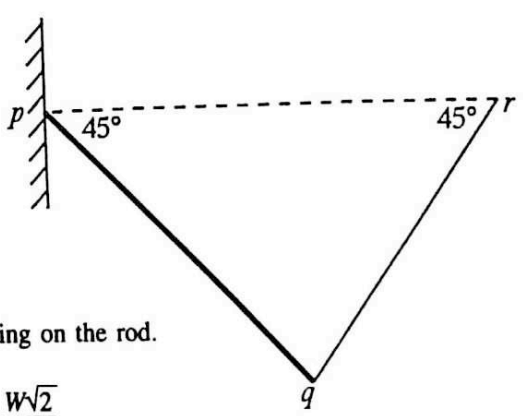
The centre of gravity of the system is at the origin.
Find the value of x and the value of y.

(b) An arrow sign is cut out from a uniform square lamina with side of length 60 cm. The sign has the line pq as an axis of symmetry. The dimensions of the sign are in cm.

Find the distance of the centre of gravity of the sign from the point p.



7. One end of a uniform rod [pq], of weight W and length 2 m, is freely hinged to a smooth vertical wall at p, while the other end q is supported by a light inextensible string tied to the point r, where p and r are on the same horizontal level. Both the rod and the string are each inclined at an angle of 45° to the horizontal.



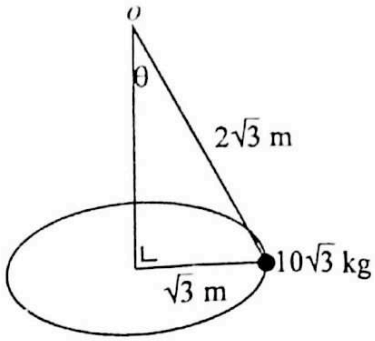
(i) Show, in a diagram, all the forces acting on the rod.

(ii) Show that the tension in the string is $\frac{W\sqrt{2}}{4}$.

(iii) Calculate, in terms of W, the horizontal and vertical components of the reaction at p. Hence, find the magnitude and direction of the reaction at p.

OVER →

8. A particle of mass $10\sqrt{3}$ kg is connected by means of a light inextensible string of length $2\sqrt{3}$ m to a fixed point o. The particle moves in a horizontal circle with radius of length $\sqrt{3}$ m and angular speed of ω rad/s.



(i) Draw a clear diagram showing all the forces acting on the particle.

(ii) Find the angle θ.

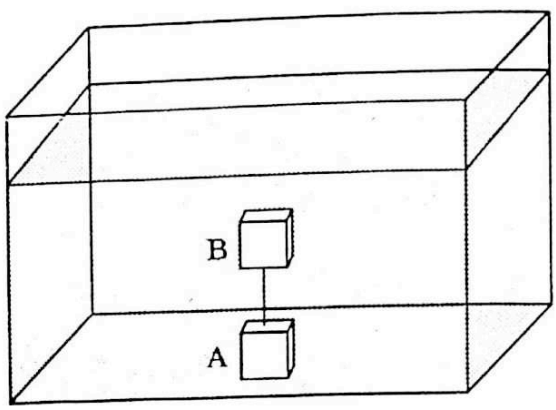
(iii) Calculate T, the tension in the string.

(iv) Find the angular speed, ω, of the particle.

9. State the principle of Archimedes.

Two cubes, A and B, each with side of length 0.5 m, are connected by a light inextensible string and placed in a tank of water so that A rests on the bottom while B floats directly above it with the string taut.

The relative density of A is 2.1 and the relative density of B is 0.8.
(Density of water = 1000 kg/m^3).



(i) Show, on two separate diagrams, all the forces acting on each cube.

(ii) Find the tension in the string.

(iii) Find the reaction between A and the bottom of the tank.