1098

M. 31

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LEAVING CERTIFICATE EXAMINATION, 1992

APPLIED MATHEMATICS — ORDINARY LEVEL

FRIDAY, 26 JUNE — MORNING, 9·30 to 12·00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 10 m/s^2 .
$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. A stone is thrown vertically upwards with a speed of 10 m/s from a point k which is 120 m above the level ground.

(i) Find the greatest height above k reached by the stone.

(ii) Find the time taken by the stone to reach the level ground.

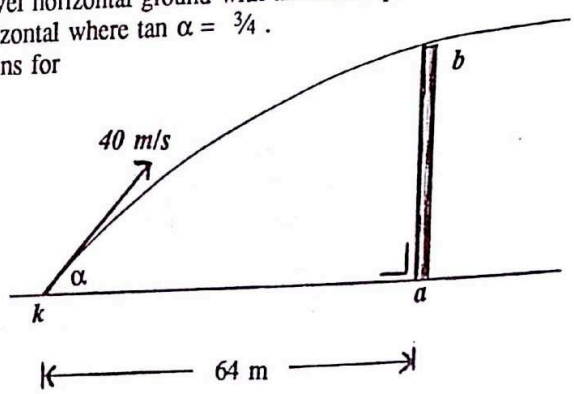
2. **(a)** The velocity of a ship relative to the current is due North at 10 km/hr. If the velocity of the current is due East at 7·5 km/hr, find the velocity of the ship.

(b) Port Q is 200 km due North of port P. The sailing time of a ship from P to Q is scheduled as 20 hours. If the velocity of the current is due East at 7·5 km/hr, find the direction and velocity with which the ship should head in order to complete the journey from P to Q in the scheduled time.

3. A particle is projected from a point k on level horizontal ground with an initial speed of 40 m/s inclined at an angle α to the horizontal where $\tan \alpha = \frac{3}{4}$.
Write down, in terms of $\vec{i}$ and $\vec{j}$, expressions for

(i) the initial velocity
(ii) the velocity after t seconds.

During its flight the particle just passes over the top of a vertical pole $[ab]$ such that $|\,ka\,| = 64$ m.
Find the height of the pole $[ab]$.



Find also the horizontal distance from k of a second vertical pole of the same height, so positioned that the particle again just passes over the top of it.

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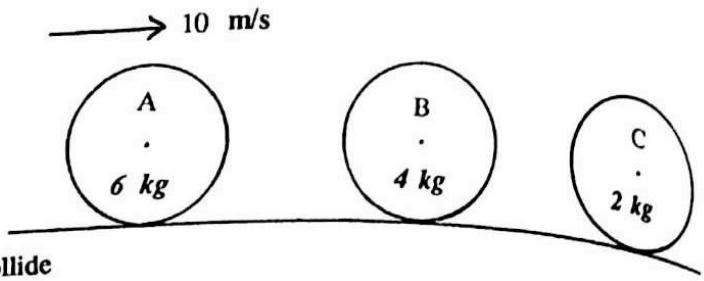
4. Three smooth spheres A, B anc C are stationary in a straight line on a smooth horizontal surface. The mass of sphere A is 6 kg, of sphere B is 4 kg and of sphere C is 2 kg.



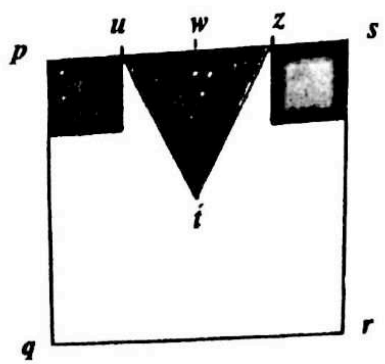
A is projected at 10 m/s so as to collide directly with B. As a result of this collision B moves forward to collide directly with C.

If the coefficient of restitution for each collision is ½,
find the final velocity of each sphere.

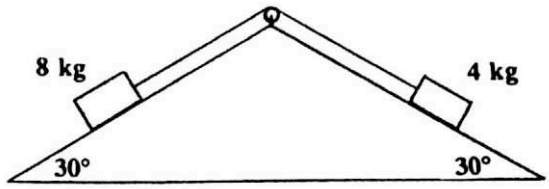
5. **(a)** Particles of mass 4 kg, 5 kg and m kg are placed on a horizontal plane at the points (⅓, 1), (3, 1) and (x, -2) respectively, so that the centre of gravity of the system is at the point (0, 0).
Calculate the value of m and the value of x.

(b) $pqrs$ is a uniform square lamina with side of length 4 cm and the diagonals of which intersect at t. The side $[ps]$ is divided into four equal parts by the points u, w and z. Three shaded pieces are cut out of the lamina, two squares each of side unit length and triangle utz.
Let q be the origin (0, 0).
For the remaining piece

(i) does its centre of gravity lie on the line qs, line pr, or line wt?
(ii) calculate the distance of its centre of gravity from qr.



6. Particles of masses 8 kg and 4 kg are held on the smooth faces of a double inclined plane and connected by means of a light inextensible string passing over a smooth pulley at the top of the plane. The faces of the plane are both inclined at 30° to the horizontal.
The system is released from rest.



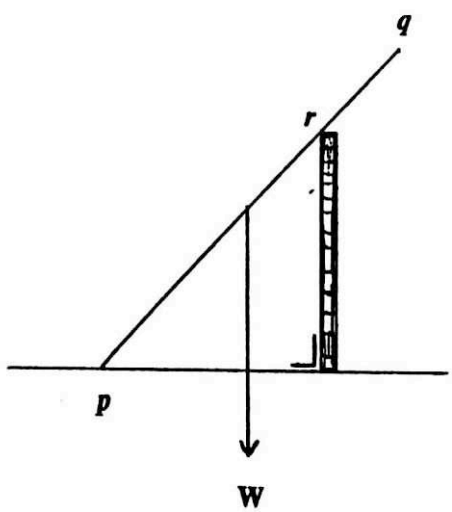
(i) Show on separate diagrams the forces acting on each particle.

(ii) Write down the equation of motion for each particle.

(iii) Find the acceleration of the system and the tension in the string.

7. A uniform ladder $[pq]$ of length $4h$ and of weight W rests against a smooth vertical wall. The point r of the ladder is in contact with the wall and the end p rests on rough horizontal ground where the coefficient of friction is μ.

The ladder is on the point of slipping when inclined at 45° to the horizontal and such that $|pr| = 3h$.



(i) Show on a diagram all the forces acting on the ladder.

(ii) Show that the normal reaction at r is $\frac{W\sqrt{2}}{3}$.

(iii) Find the normal reaction at p in terms of W.

(iv) Hence prove that $\mu = \frac{1}{2}$.

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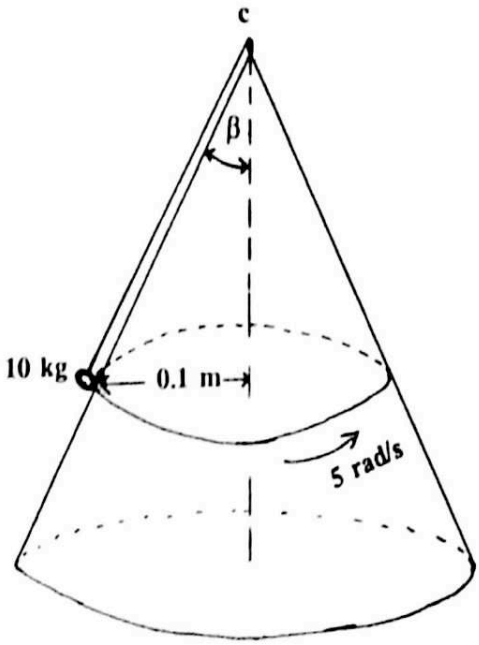
8. A particle of mass 10 kg is connected by means of a light inextensible string to a fixed point c which is vertically just above the apex of a right circular cone whose semi-vertical angle is β where $\tan \beta = \frac{3}{4}$.



The particle describes a horizontal circle of radius 0·1 m on the smooth outside surface of the cone. The constant angular speed of the particle is $\omega = 5$ rad/s.

(i) Show in a diagram the three forces acting on the particle.

(ii) Show that the value of the normal reaction between the particle and the cone is 40 N.

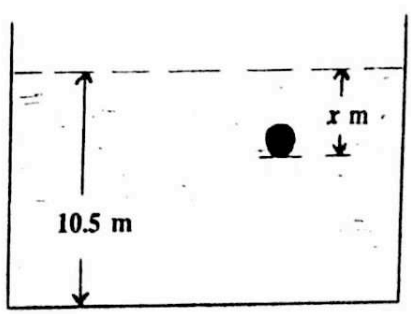
(iii) Calculate the tension in the string.

9. State the Principle of Archimedes.

A uniform solid sphere whose mass is 60 kg is made of a material whose relative density is 1·5. Find the volume of the sphere.

If the sphere is completely submerged in water (density = 1000 kg/m^3), what is the upthrust on the sphere due to the water?

A tank contains water to a depth of 10·5 m. The sphere is held at rest in the water at a distance of x m from the surface of the water.



The sphere is released and moves downwards. If it takes 2 s for the sphere to reach the bottom of the tank, find the value of x.