AN ROINN OIDEACHAIS **M.51**

LEAVING CERTIFICATE EXAMINATION, 1991

APPLIED MATHEMATICS - ORDINARY LEVEL

FRIDAY, 21 JUNE - MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.
Take the value of g to be 10 m/s^2.
$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively.

1. A cage goes down a mine shaft 675 m deep in 45 s. The cage starting from rest accelerates uniformly for the first quarter of the distance, taking t_1 seconds. For the last quarter of the distance, the cage decelerates uniformly to rest, again taking t_1 seconds. While traversing the central portion of the shaft, the cage travels at a uniform speed of v m/s.

(i) Draw an accurate velocity-time graph of the motion.

(ii) From the graph, or otherwise, write down two expressions for distance involving v and t_1.

(iii) Calculate the value of t_1.

(iv) Calculate the value of v.

(v) Find the acceleration of the cage.

2. A boat travels with a speed of 5 m/s in still water. The boat is required to go straight across a river at right angles to the parallel banks. The river is 120 m wide and is flowing at 3 m/s parallel to the straight banks.

On a diagram, show the velocity of the boat, velocity of the river and the velocity of the boat relative to the river. At what angle to the bank should the boat head ?

Find the velocity of the boat relative to the river.

How long does it take the boat to cross the river ?

On the return journey, the boat heads straight across the river and as a result is carried downstream by the current.

Find for the return journey of the boat

(i) the distance travelled.

(ii) the time taken.

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3. A particle is projected from a point k on level horizontal ground with an initial speed of 26 m/s inclined at an angle θ to the horizontal where $\tan\theta = \frac{5}{12}$.

Write down, in terms of $\vec{i}$ and $\vec{j}$, expressions for

(i) the initial velocity.

(ii) the velocity after t seconds.

Calculate the maximum height attained by the particle.

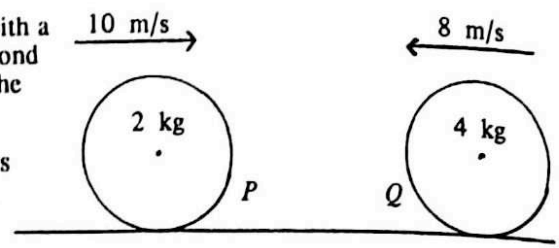
A second particle is projected from the point k at an angle of $30°$ to the horizontal. It reaches its maximum height in the same amount of time as it took the first particle to attain its maximum height.

Calculate the initial speed of projection of the second particle.

4. A smooth sphere P of mass 2 kg moving with a speed of 10 m/s collides directly with a second smooth sphere Q of mass 4 kg moving in the opposite direction with a speed of 8 m/s.

Due to the collision the speed of sphere P is reversed in direction and reduced to 8 m/s.

Find

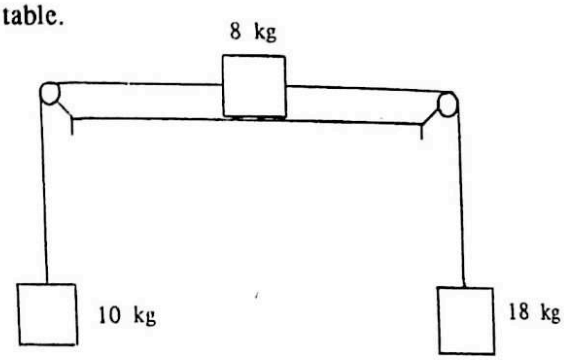


(i) the speed of sphere Q after the collision

(ii) the value of the coefficient of restitution.

After this collision, sphere Q goes on to collide directly with a third smooth sphere R, of mass 8 kg, which is at rest. As a result of this collision, sphere Q is brought to rest. Show that the coefficient of restitution for this collision is the same as that for the first. Find the total loss in kinetic energy due to the two collisions.

5. A particle of mass 8 kg lies on a rough horizontal table. The coefficient of friction on this table is ½. This mass is connected by two light inelastic strings passing over smooth pulleys at opposite edges of the table to two masses of 18 kg and 10 kg which hang freely as in diagram. The system is released from rest so that the 18 kg mass moves downwards.



(i) Show on separate diagrams the forces acting on each particle. Hence calculate the size of the friction force on the 8 kg mass.

(ii) Write down the equation of motion for each particle.

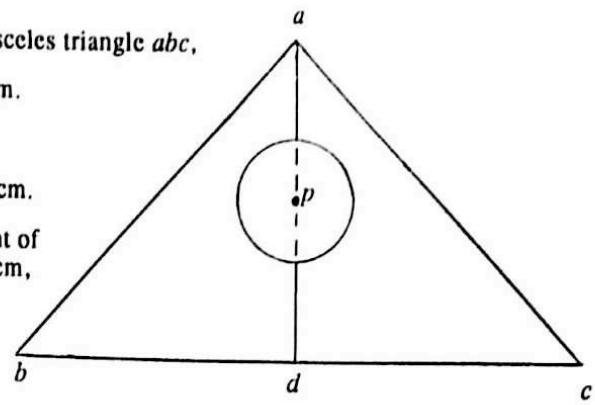
(iii) Find the acceleration of the system and the tensions in the two strings.

6. (a) Particles of weight 40 N, 30 N and 20 N are placed on a horizontal plane at the points $(1, 2)$, $(x, -1)$ and $(-3, y)$ respectively, so that the centre of gravity of the system is at the point $(0, 0)$. Calculate the value of x and the value of y.

(b) A uniform lamina is in the shape of an isosceles triangle abc, where $|ab| = |ac|$ and $|bc| = 9$ cm.

The perpendicular height is $|ad| = 6$ cm.

A disc, whose centre is at p the mid-point of $[ad]$ and whose radius is of length 0·7 cm, is cut out from the lamina.



Find the distance of the centre of gravity of the remaining piece from bc.
Give your answer correct to two decimal places.

Take $\pi = \frac{22}{7}$.

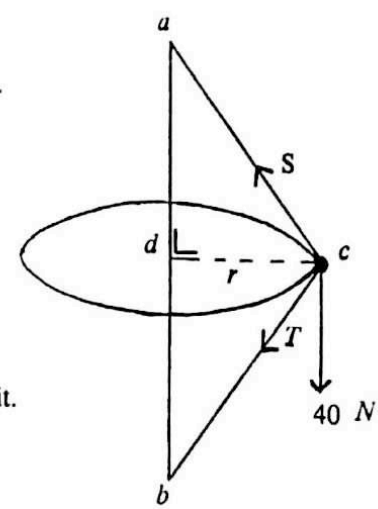
7. A particle of mass 4 kg is attached to two points a and b on a vertical wire by means of two light inelastic strings $[ac]$ and $[bc]$ each of length 7·5 m. The wire $|ab| = 12$ m.

The particle describes a horizontal circle with centre d, the mid-point of $[ab]$.

The angular speed, ω, of the particle is $\sqrt{5}$ rad/s.

S and T are the tensions in the two strings.

The diagram shows the particle and the forces acting on it.



(i) Find r, the length of the radius of the horizontal circle.

(ii) Calculate the centripetal force.

(iii) Write down two expressions for the tensions in the two strings.

(iv) Find the value of S and the value of T.

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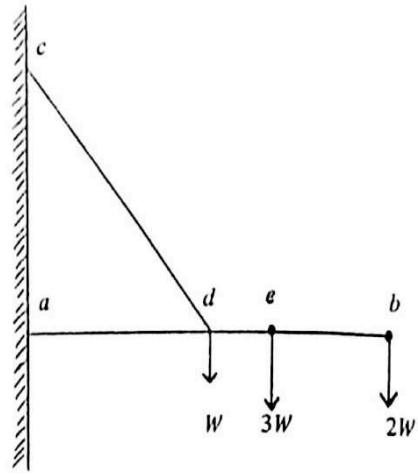
8. A uniform beam [ab] of length 12 m and weight W is freely jointed to a vertical wall ac. The beam is held in a horizontal position by a string attached to the midpoint d of the beam and to a point c on the wall such that $|cd| = 10$ m. Particles of weight $2W$ and $3W$ are attached to the beam at b and e, respectively, where $|ae| = 8$ m.



(i) Show on a diagram all the forces acting on the beam.

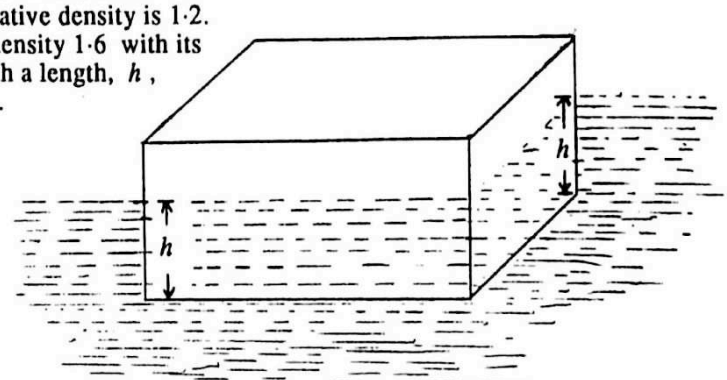
(ii) If X is the horizontal component and Y the vertical component of the reaction at the hinge, write down an expression involving X and an expression involving Y.

(iii) By taking moments about a suitable point, find a value for Y in terms of W.

(iv) Hence express the tension in the string in terms of W and also find a value for X in terms of W.

9. State the principle of Archimedes.

A uniform rectangular block, 0·5 m x 0·2 m x 0·2 m is made of a material whose relative density is 1·2. It floats in a liquid of relative density 1·6 with its longest edge horizontal and with a length, h, of its vertical edges submerged.



Find

(i) the weight of the block

(ii) the upthrust on the block due to the liquid in terms of h

(iii) the value of h.

A piece of iron is next placed on the block so that the block is totally submerged but the iron remains just above the surface of the liquid.

What is the weight of the piece of iron ?

The piece of iron is then weighed in water and found to have a weight of 70 N in water. Find the relative density of the iron ?

(Density of water = 1000 kg/m^3).