LEAVING CERTIFICATE EXAMINATION, 1990

APPLIED MATHEMATICS - ORDINARY LEVEL

FRIDAY, 22 JUNE - MORNING, 9.30 - 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.
Take the value of g to be 10 m/s^2.
$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively.

1. Two points, p and q, are some distance apart on a straight road.
A car starts from rest at p and accelerates uniformly for 10 s to its maximum speed of 20 m/s. It maintains this speed for 20 s and then decelerates to rest at q in a further 6 s.

   (i) Draw an accurate velocity-time graph of the motion.

   (ii) Find the distance from p to q.

   (iii) Find the acceleration and deceleration of the car.

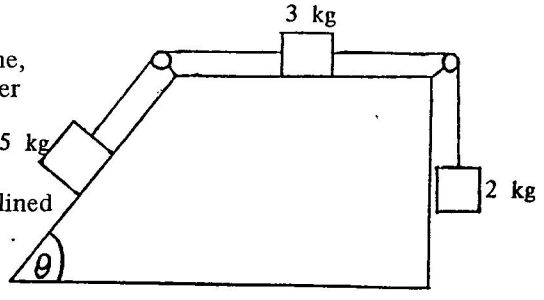
If the car, after reaching its maximum speed, maintained this speed until it passed q, find the total time taken for the car to travel from p to q.

2. At noon ship **P** is 16 km due East of ship **Q**. Ship **P** is sailing with velocity $12\vec{i} - 5\vec{j}$ km/hr and ship **Q** is sailing with velocity $20\vec{i} - 5\vec{j}$ km/hr.

Find, in terms of $\vec{i}$ and $\vec{j}$, the velocity of **Q** relative to **P**. Explain why **Q** will intercept **P**. Find the time taken for **Q** to catch **P**.

If, however, at noon **P** changed its velocity to $12\vec{i} + \vec{j}$ km/hr while **Q**'s velocity remained the same, what would be the shortest distance between the ships in this instance ?

3. A particle of mass 3kg, on a smooth horizontal plane, is connected by two light inelastic strings passing over smooth pulleys at opposite edges of the plane, to two particles of masses 5 kg and 2 kg.



The particle of mass 5 kg is on a smooth plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$.

The particle of mass 2 kg hangs freely.
The system is released from rest.

(i) Show, on separate diagrams, the forces acting on each particle.

(ii) Write down the equation of motion for each of the three particles.

(iii) Hence find the acceleration of the particles and the tensions in the two strings.

4. A projectile is fired from a point h on horizontal ground with an initial speed of 50 m/s inclined at an angle α to the horizontal where $\tan\alpha = \frac{4}{3}$.

Write down, in terms of $\vec{i}$ and $\vec{j}$, expressions for

(i) the initial velocity

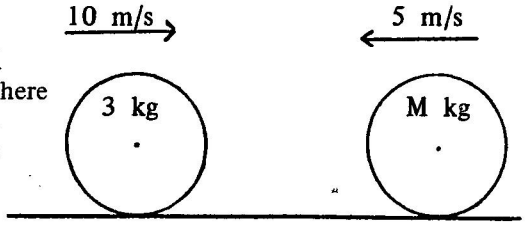
(ii) the displacement from h after t seconds

(iii) the velocity after t seconds.

Find the time of flight and the range.

Find, also, the values of t for which the projectile is at a vertical height of 60 m above the ground.

5. A smooth sphere of mass 3 kg moving with a speed of 10 m/s collides directly with a second smooth sphere of mass **M** kg moving in the opposite direction with a speed of 5 m/s. As a result of the collision the sphere of mass **M** kg is brought to rest.



If the coefficient of restitution between the spheres is $\frac{1}{3}$, find

(i) the speed of the 3 kg mass after the collision

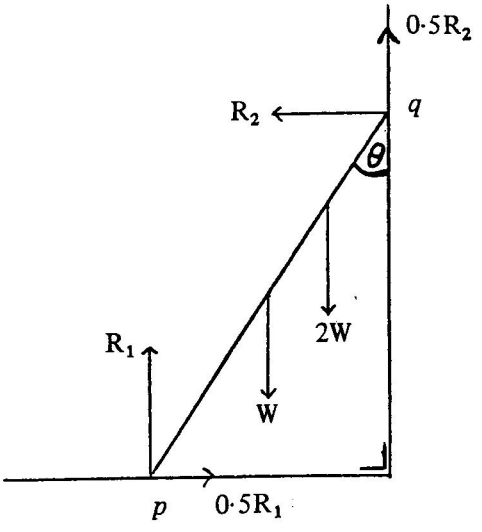
(ii) the mass, M kg, of the second sphere

(iii) the loss in kinetic energy due to the collision

(iv) the impulse imparted to the 3 kg sphere.

6. A uniform ladder, $[pq]$, of weight W and length 4h stands with the end p on rough horizontal ground and the end q against a rough vertical wall. The coefficient of friction is the same at p and q and is equal to 0·5.

A weight, 2W, is fixed to the ladder at a distance h from the end q so that the ladder is on the point of slipping when inclined at an angle θ to the vertical, as shown.



The diagram shows all the forces acting on the ladder.

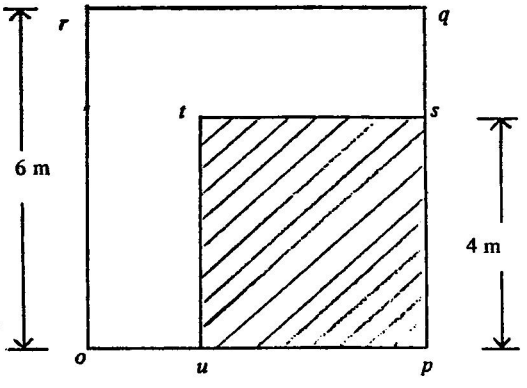
(i) Express the normal reactions R_1 and R_2 at p and q in terms of W.

(ii) **By taking moments about p, show that**

$$\tan\theta = \frac{6}{7}.$$

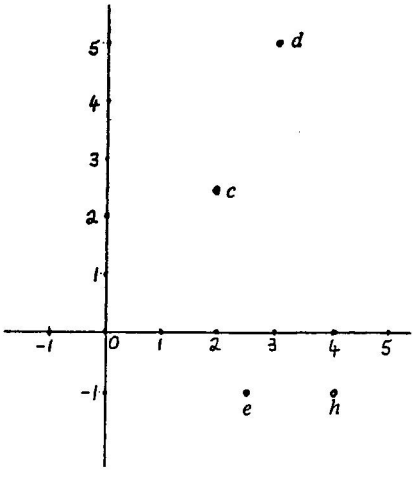
7. (a) $opqr$ is a uniform square lamina of side 6 m in length and of mass 36 kg. A uniform square, $upst$, of side of length 4 m and of mass 16 kg is cut from the lamina as shown.

Taking o as origin, find the coordinates of the centre of gravity of the remaining piece.

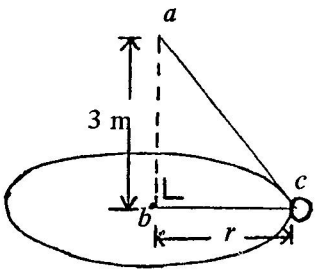


(b) Particles of weight 4N, 5N, 6N and 7N are at points $c(2, 2{\cdot}5)$, $d(3, 5)$, $e(2{\cdot}5, -1)$ and $h(4, -1)$.

Find the centre of gravity of the system.



8. The ends a and b of a light inelastic string acb are fixed to a vertical wire.



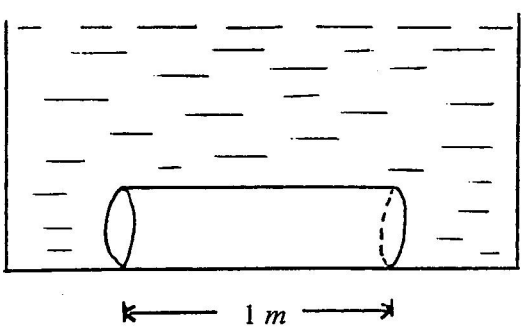
A ring, of mass 0·6 kg, is attached to the string at the point c. The ring is made to describe a horizontal circle of radius r with centre b which is 3 m vertically below a.

If $|ac| = 5$ m, calculate r.

Draw a diagram to show all the forces acting on the ring. Calculate T, the tension in the string. Show that the angular speed, ω, of the ring is given by

$$\omega = \sqrt{7{\cdot}5} \text{ rad/s.}$$

9. A uniform solid cylinder is held at rest in a horizontal position at the bottom of a tank of fresh water (density = 1000 kg/m^3).



The relative density of the material of the cylinder is 0·25. The radius of the cylinder is 0·2 m and its length is 1 m.

Find, taking $\pi = 3$,

(i) the volume of the cylinder
(ii) the weight of the cylinder
(iii) the upthrust on the cylinder due to the water
(iv) the net upward acceleration of the cylinder when it is released.