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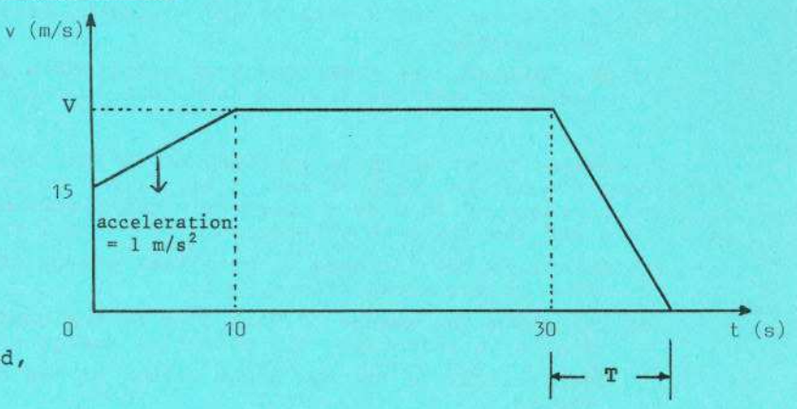
LEAVING CERTIFICATE EXAMINATION, 1989

APPLIED MATHEMATICS - ORDINARY LEVEL

FRIDAY, 23 JUNE - MORNING, 9.30 - 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.
Take the value of g to be 10 m/s^2.

1. The velocity-time graph shown describes the general features of motion of a cyclist as she travels a total distance of 825 m. The acceleration in the first part of the journey is 1 m/s^2.



(i) Find V, the maximum speed of the cyclist.

(ii) Use your graph to calculate the time T, as shown in the diagram.

(iii) Hence find the total time taken for the journey.

(iv) Calculate the cyclist's average speed, in m/s, during the journey.

2. A boat A is sailing with speed 2 m/s in a direction 15° North of East, and a boat B is sailing with speed 2 m/s in a direction 15° East of North.

(i) On a diagram, show the velocity of A, the velocity of B, and the velocity of B relative to A. Show clearly the directions North and East.

(ii) Calculate the size and direction of the velocity of B relative to A.

(iii) At 10.00 a.m., B is 1200 m due East of A.
On a second diagram, show the position of A and the position of B at 10.00 a.m., and the relative path of B with respect to A.

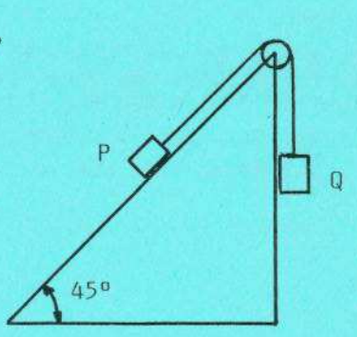
(iv) Hence calculate the shortest distance between A and B.

3. A projectile is fired from a point k on level horizontal ground with initial velocity $a\,\mathbf{i} + b\,\mathbf{j}$ m/s, where $\mathbf{i}$ and $\mathbf{j}$ are unit perpendicular vectors pointing along the horizontal and upward verticals, respectively. 3·5 seconds later the projectile just clears an obstacle whose displacement from k is $105\,\mathbf{i} + 113.75\,\mathbf{j}$ m.

(i) Find the value of a and the value of b.

(ii) Hence find the displacement of the projectile 2 seconds after it has just cleared the obstacle.

4. A mass P of $2\sqrt{2}$ kg lies on a rough plane which is inclined at 45° to the horizontal. The coefficient of friction on this plane is ½. The mass P is connected by means of a light string, which passes directly up the plane and over a smooth pulley, to a second mass Q of 4 kg, which hangs freely as in diagram. When the system is released from rest the downward acceleration of Q is a m/s^2.



(i) Show in separate diagrams the forces acting on P and on Q.

(ii) Calculate the normal reaction of the rough plane to P.

(iii) Hence calculate the size of the friction force on P.

(iv) Write equations of motion for each of P and Q.

(v) Calculate the tension in the string if a = 1·5 m/s^2.

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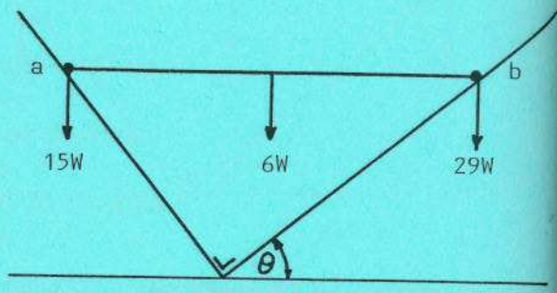
5. A uniform metre stick $[ab]$, of weight $6W$, has weights $15W$ and $29W$ attached to it at a and b, respectively, as in diagram. The metre stick rests horizontally in a smooth-sided right-angled frame with b against the side inclined at an angle θ to the horizontal with $\cos\theta = \frac{4}{5}$.



(i) By taking moments about a suitable point, locate the resultant force R of the three forces $6W$, $15W$ and $29W$.

(ii) Show in a diagram the force R and the normal reactions acting on the stick.

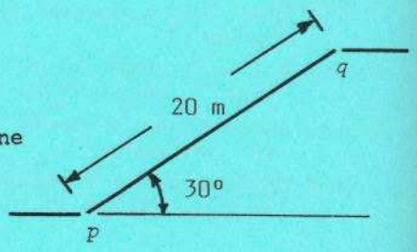
(iii) Show that the normal reaction at a is $30W$.

6. A small smooth sphere A, of mass 3 kg, is moving at 4 m/s when it collides directly with a second small smooth sphere B, of mass 2 kg, which is moving in the same direction at 2 m/s.

(i) If spheres A and B collide and coalesce, find the speed with which the compound body moves.

(ii) If, instead, the coefficient of restitution for the collision is $\frac{1}{4}$, find the speeds of spheres A and B after the collision.

7. A conveyor belt, which is inclined at 30° to the horizontal, lifts concrete blocks, each of mass 6 kg, from p to q, an inclined distance of 20 m as in diagram. The blocks move at 0·5 m/s and the distance between each block is 1 m.



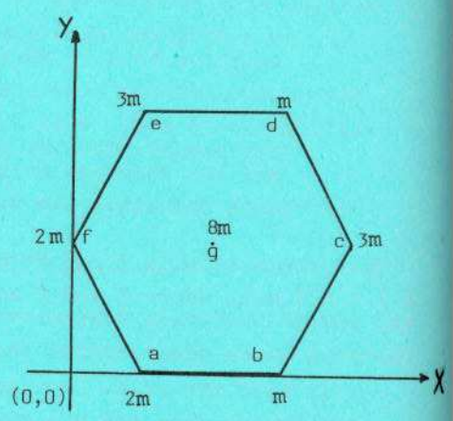
(i) Calculate the increase in potential energy of each block when it has moved from p to q.

(ii) Calculate the amount of work done by the conveyor belt in one minute in increasing the potential energy of the blocks.

(iii) Hence calculate the net power output of the conveyor belt engine.

(iv) If the speed of the belt is increased so that the kinetic energy of each block on the belt becomes 3 J, find the new speed with which each block moves.

8. A uniform regular hexagonal lamina $abcdef$, of side 10 cm and of mass 8 m, has masses of 2m, m, 3m, m, 3m and 2m at vertices a, b, c, d, e and f, respectively. The lamina is arranged so that ab lies along the X-axis as in diagram.



(i) Calculate the coordinates for each of the vertices given that the coordinates of the centre g of the hexagon are $(10, 5\sqrt{3})$.

(ii) Calculate the x- and y- coordinates of the centre of mass of the entire system.

9. A body of mass 0·088 kg is held on the end of a spring. When the body is lowered into fresh water (density = 1000 kg/m^3) the extension of the spring indicates an apparent weight of 0·08 N for the body.

(i) Calculate the upthrust on the body in fresh water.

(ii) Hence find the volume and the density of the body.