AN ROINN OIDEACHAIS

M.51

LEAVING CERTIFICATE EXAMINATION, 1988

APPLIED MATHEMATICS - ORDINARY LEVEL

FRIDAY, 17 JUNE - MORNING, 9.30 - 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.
Take the value of g to be 10 m/s^2.

1. A train starting from rest leaves a station and accelerates uniformly. It passes in succession three telegraph poles spaced 100 m apart. The times between the first and second poles and between the second and third poles are 12 seconds and 8 seconds, respectively.
   Find:-
   (i) the acceleration of the train;
   (ii) the distance of the station from the first pole.

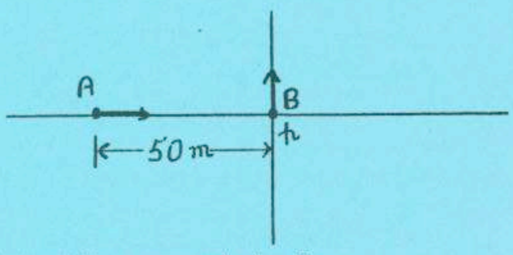
2. (a) A uniform beam 3 m long, with a mass of 25 kg hanging from one end balances in a horizontal position on a support about a point 0·9 m from this end.
   Find:-
   (i) the mass of the beam,
   (ii) the pressure on the support.

   (b) A uniform rod, of length 12 m and of mass 48 kg is bent so as to form three sides of a square.
   Find the centre of gravity of this shape.

3. A particle is fired upward and out to sea with speed 13 m/s at an angle α to the horizontal from a point on top of a cliff 60 m high above the sea. If $\tan\alpha = \frac{5}{12}$, use horizontal and vertical components of velocity and acceleration to:-
   (i) find the magnitude to the nearest whole number and also the direction of the velocity of the particle after 2 s,
   (ii) calculate how long after its projection the particle takes to reach a position of 41·25 m above the sea.

4. A motor car, of total mass 1500 kg, is running on a level road at a uniform speed of 48 km/h. On reaching a decline, which descends at a uniform gradient of 1 in 20, it is allowed to free-wheel and the speed is observed to remain the same as before.
   Draw a diagram showing all the forces acting on the car as it descends the decline and
   (i) calculate the resistance to the motion of the car.
   (ii) find the power at which the engine of the car is working on the level road, the resistance being the same as in (i).

5. Two roads cross at right angles at p as shown. A person, A, is walking along one of them at 4·5 km/h. Another person, B, walking on the other road at 6 km/h is at p when A is 50 m from p.



   (i) In a diagram, show the relative velocity of A with respect to B.
   (ii) Calculate the magnitude and direction of this relative velocity.
   (iii) Show that A and B will be closest together after $\frac{72}{5}$ seconds.
   (iv) Hence find how far A has walked during this time.

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6. A particle x, of mass m_1, is attached to a fixed point o by a light inextensible string. This particle describes a horizontal circle beneath o with p as centre and with angular velocity ω_1. The angle pox measures 60°.

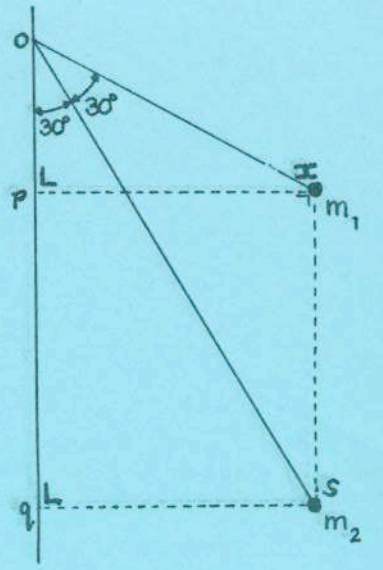
A second particle s, of mass m_2, is also attached to the fixed point o by another light inextensible string. Particle s also describes a horizontal circle beneath o with q as centre and with angular velocity ω_2. The angle qos measures 30°.

The radii of both circles, $|px|$ and $|qs|$, are equal.

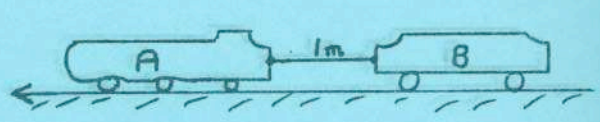
(i) Show in separate diagrams the forces acting on each particle.

(ii) Express the tension in each of the strings in terms of their respective masses.

(iii) Show that the ratio $\frac{\omega_1}{\omega_2} = \sqrt{3}$.



7. An engine A of mass 270 tonnes is coupled by means of a taut chain, 1 m long, to a carriage B of mass 130 tonnes. The engine and carriage are moving with constant speed 10 m/s when a braking force of 240 kN is applied to engine A. As a result the engine decelerates, the chain becomes slack and the carriage B catches up to collide with the engine.



(i) Calculate the deceleration of engine A before the collision.

(ii) If after the braking force is applied engine A travels a distance x m before the collision, express in terms of x the distance travelled by carriage B.

(iii) Find the length of time after the braking force is first applied until the carriage B and engine A collide.

(iv) Calculate the speed of engine A as carriage B collides with it.

(v) If the coefficient of restitution for the collision was zero, find the speed of engine A after the collision.

8. A piece of wood of mass 35 g and relative density 0·7 is held at rest at the bottom of a deep tank of salt water of relative density 1·05.

(i) Find the volume of the wood.

(ii) Calculate the upward force, in Newtons, on the wood due to the salt water.

(iii) Find the net upward acceleration of the wood when it is released.

9. A sphere Z with centre o and of weight W rests on two parallel half cylinders X and Y, with centres k and s, respectively, which in turn rest on a rough table as shown.

The points of contact of the sphere with X and Y are a and b, respectively. The radii $[oa]$ and $[ob]$ make angles of 45° and 30°, respectively, with the downward vertical.

(i) Show in a diagram the three forces acting on Z.

(ii) Let R_1 and R_2 be the reactions at a and b, respectively. By resolving these horizontally, show that

$$R_1 = \frac{R_2}{\sqrt{2}}.$$

Hence find the value of R_1 as a surd in terms of W.

(iii) If the half cylinder X is of weight $\frac{W}{1+\sqrt{3}}$ and is about to slide on the table, show that the coefficient of friction for the surface is ½.