AN ROINN OIDEACHAIS

M.51

4370

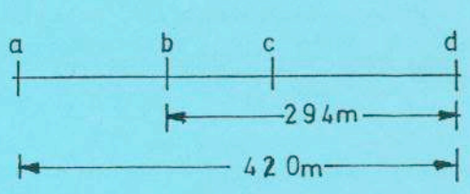
LEAVING CERTIFICATE EXAMINATION, 1987

APPLIED MATHEMATICS - ORDINARY LEVEL

MONDAY, 29 JUNE - AFTERNOON, 2.00 - 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.
Take the value of g to be 10 m/s^2.

1. An engine is travelling at full speed of 30 m/s on a level straight track. When the engine is at a the driver sees a red signal at d, 420 m ahead of him. He immediately applies the brakes and the engine decelerates at 3 m/s^2. At b, which is 294 m from d, the engine stops decelerating and travels at constant speed for 7 seconds until it reaches c. At that time the signal changes to green and the engine immediately accelerates to just 30 m/s again as it passes d.



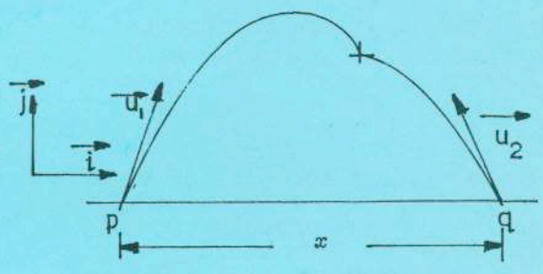
(i) Draw a rough velocity-time graph for the motion of the engine between a and d.

(ii) Find the speed of the engine between b and c.

(iii) Calculate the distance from c to d and hence find the acceleration of the engine between c and d.

2. At a certain instant a projectile A is fired from a point p on level horizontal ground with initial velocity $\vec{u}_1 = 9\vec{i} + 30\vec{j}$ m/s, where $\vec{i}$ and $\vec{j}$ are unit vectors pointing along the horizontal and the upward vertical, respectively.

Three seconds later a second projectile B is fired from a point q on the ground with initial velocity $\vec{u}_2 = -14\vec{i} + 45\vec{j}$ m/s. The displacement of q from p is $x\vec{i}$ m. The two projectiles collide t seconds after B is fired.



(i) Write down the displacement of A after $(t + 3)$ seconds.

(ii) Write down the displacement of B after t seconds.

(iii) By comparing the $\vec{j}$-components of the displacement of each projectile at impact, find the value of t.

(iv) Calculate the value of x.

3. A football player A gets the ball at a point n and runs northwards with it at a speed of 6 m/s. At the same instant an opponent B starts at a speed of 6·5 m/s from a point h due west of n to try to tackle A.

(i) What is the velocity of B relative to A ?

(ii) If $|nh| = 12{\cdot}5$ m, how long will B take to reach A ?

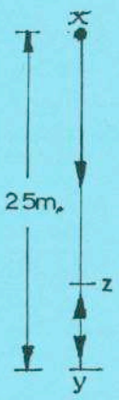
(iii) How far has B to run before tackling A ?

4. A ball of mass 2 kg drops through the air from rest at a point x, at a height of 25 m above the ground. The ball hits the ground at y with a speed of 20 m/s.

Find how much work has been done by the air in resisting the movement of the ball. Find also the fraction of the initial total energy lost through air resistance.

If the coefficient of restitution at y is 0·5, calculate the speed at which the ball rebounds.

The ball then rises as far as z, a height h above the ground. If the same fraction of total energy as before is lost through air resistance, find the value of h.



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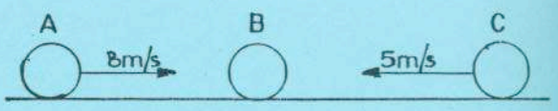
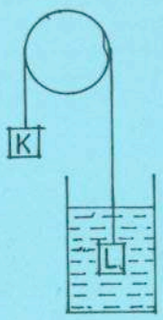
5. Three equal smooth spheres A, B, C are in a straight line on a smooth horizontal surface. The mass of each sphere is 1 kg. A is moving at 8 m/s towards B, which is stationary. C is on the far side of B and is moving at 5 m/s, also towards B. Spheres A and B collide and coalesce. Find the speed with which the compound body moves.



This compound body is then brought to rest by a collision with sphere C. Find the coefficient of restitution for this collision.

6. Two masses K and L are connected by a light inextensible string passing over a smooth fixed pulley. L is completely immersed in water and the system is held at rest.
The mass of K is 4·5 kg and the mass of L is 7·5 kg. The relative density of L is 1·5. The water offers a resistance of 5 N to any movement of L.



The system is now released from rest. Show and label in separate diagrams the two forces acting on K and the four forces on L. Calculate the upthrust on L.

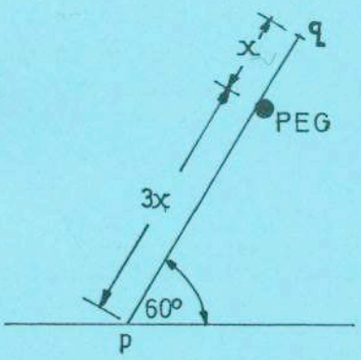
Show that the upward acceleration of L when released is 1.25 m/s^2.

7. A uniform rod $[pq]$, of weight W and length $4x$, rests against a fixed smooth peg a distance $3x$ from p. The end p rests on a rough horizontal surface. The rod makes an angle of 60° with the surface.
Show in a diagram the four forces acting on the rod.
Express, in terms of W, the normal reaction at:-

(i) the peg

(ii) the end p.

Show that the coefficient of friction, when the rod is on the point of slipping at p, is $\frac{\sqrt{3}}{5}$.



8. A uniform rod $[ab]$ of weight 50 N and of length 10 m is in equilibrium resting on two supports situated at s and v. s is 1 m from a and v is 2 m from b. Two other external forces act on the rod:

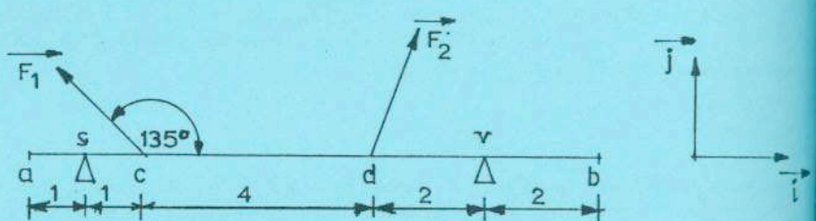
$\vec{F}_1$ is $10\sqrt{2}$ N at 45° to ba, acting at c, where $|ac| = 2$ m.

$\vec{F}_2$ is $p\vec{i} + 17\vec{j}$ N, acting at d, where $|cd| = 4$ m, and where $\vec{i}$ and $\vec{j}$ are unit perpendicular vectors acting along ab and perpendicular to ab, respectively.

(i) Convert $\vec{F}_1$ to $\vec{i}$ and $\vec{j}$ form.

(ii) Calculate the magnitude of the normal reactions at s and v.

(iii) Write down the value of p.



9. Using the X and Y axes, three points are identified as $a(0,0)$, $b(0,6)$ and $c(8,0)$.

(i) Weights of 7 N, 6 N and 3 N are located at a, b and c, respectively; Find the centre of gravity of the three weights.

(ii) A uniform wire is bent into the shape of the triangle abc. Each unit length of the wire weighs 1 N. Find the centre of gravity of the wire frame.