AN ROINN OIDEACHAIS

M.51

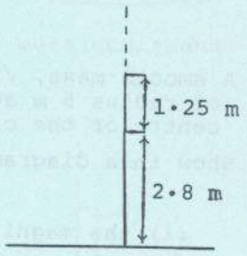
LEAVING CERTIFICATE EXAMINATION, 1986

APPLIED MATHEMATICS - ORDINARY LEVEL

FRIDAY, 27 JUNE - AFTERNOON, 2.00 - 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.
Take the value of g to be 10 m/s^2.

1. A particle falls from rest and eventually strikes the ground. During the fall, it traversed 1·25 m in 0·1 s, where this 1·25 m was 2·8 m above ground level.



Calculate the height of the total fall and the average speed during this fall.

2. A helicopter pilot wishes to fly from p to q as quickly as possible. q is 130 km north of p. The maximum speed of the helicopter in still air is 122 km/h. A steady wind of $22\vec{i} + 10\vec{j}$ km/h is blowing throughout the journey, where $\vec{i}$, $\vec{j}$ are perpendicular unit vectors East and North, respectively.

(i) Draw a diagram to show in what direction the helicopter must point relative to the wind and p, q.

(ii) Find the speed of the helicopter.

(iii) Calculate the time from p to q.

3. A projectile is fired across level ground from a point 40 m above ground.

Its initial velocity is $11\vec{i} + 10\vec{j}$ m/s, where $\vec{i}$ and $\vec{j}$ are unit vectors parallel and perpendicular to the ground, respectively.

Calculate the maximum height above level ground attained by the projectile and the velocity in terms of $\vec{i}$ and $\vec{j}$ with which it strikes level ground.

4. A skier slides down a slope, inclined to the horizontal at 45°, without using the sticks for acceleration. Passing points a and b on the slope, the skier's speed was 5 m/s and 25 m/s respectively and $|ab| = 60\sqrt{2}$ m. The mass of the skier and equipment was 80 kg. For the slide from a to b, calculate

(i) the gain in kinetic energy.

(ii) the loss in potential energy.

(iii) the work done overcoming friction.

Show in a diagram the forces acting on the skier and calculate the normal reaction of the slope to the skier. Verify that the coefficient of friction on the slope is $\frac{1}{2}$.

5. An engine of mass 225 tonnes is driven up an incline of 1 in 50 at a steady speed of 20 m/s when the engine exerts its maximum tractive force. The frictional force opposing motion is 10 kN.

Draw a diagram showing all the forces acting on the engine and determine

(i) the maximum tractive force exerted by the engine.

(ii) the power output of the engine.

Assuming that the frictional force remained constant at 10 kN and the engine exerts its maximum tractive force, determine the acceleration of the engine when it travels down the same incline.

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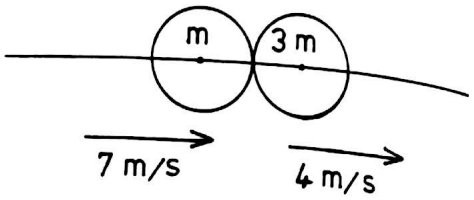
6. Spheres of mass m and $3m$ respectively, moving in the same direction in the same straight line with speeds of 7 m/s and 4 m/s, respectively, collide. If the speed of the smaller mass is reduced to 4 m/s in the same direction, find, e, the coefficient of restitution for the collision.



Assume the same initial speeds as before but the opposite direction for the speed of the lighter mass before collision and using the value of e obtained above, find the speed of each mass after collision.

7. A smooth mass, $\sqrt{3}$ kg, is placed on the smooth inside of a hemispherical bowl of radius 5 m and it moves in a horizontal circle at uniform speed. The centre of the circle is 2·5 m below the centre of the circle formed by the rim of the bowl.

Show in a diagram the forces acting on the mass and calculate

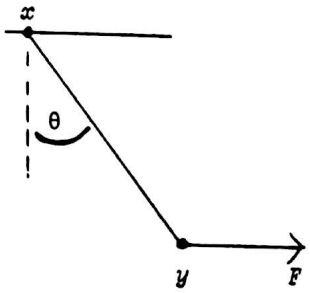
(i) the magnitude of the normal reaction of the bowl to the mass.

(ii) the angular velocity of the mass.

(iii) the velocity of the mass.

(iv) the magnitude of the centripital force.

8. (a) A uniform rod $[xy]$ of weight W and length $2l$ is smoothly hinged at x to a fixed point in a horizontal ceiling. A horizontal force F is applied at y until xy is in equilibrium at an angle θ to the vertical and $\tan\theta = \frac{1}{2}$.



Express in terms of W

(i) the force F.

(ii) the horizontal and vertical components of the reaction of the hinge.

If the horizontal force at y is then doubled, find the value of θ_1, the angle the rod then makes with the vertical.

9. A uniform wooden cylinder is in equilibrium in a bath of water with its axis vertical and nine tenths of its volume submerged. The wood is held down by a vertical string tied from the midpoint of its base to the base of the bath. If the densities of wood and water are 600 kg/m^3 and 1000 kg/m^3, respectively, and the tension in the string is 15N, calculate

(i) the volume of the wood.

(ii) the mass of the wood.