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LEAVING CERTIFICATE EXAMINATION, 1985

APPLIED MATHEMATICS - ORDINARY LEVEL

FRIDAY, 28 JUNE - AFTERNOON, 2.00 - 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.

1. A particle started from rest and accelerated uniformly at 3 m/s^2 until a speed of 21 m/s was reached. This speed was maintained for 8 s and then the particle was brought to rest in 3 s by uniform deceleration.

   Calculate (i) the number of seconds to reach the speed of 21 m/s
   (ii) the deceleration.

   Draw a velocity-time graph for the motion, and hence, or otherwise, find the total distance travelled.

2. A ship, A, sailed due East at 12 m/s and another ship, B, sailed due North at 16 m/s. At noon A was 1 km West of B.

   (i) In a diagram, show the relative velocity of A with respect to B.
   (ii) Calculate the magnitude and direction of this relative velocity.
   (iii) Draw a diagram of the situation at noon and on it show the relative path of A with respect to B.
   (iv) Find the shortest distance between A and B.

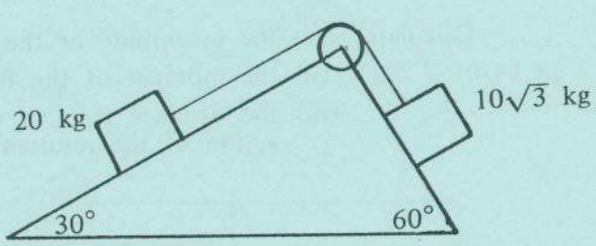
3. A particle projected from o, on level ground, with initial speed of 20 m/s at an angle of 30° to the horizontal, strikes the ground at a point p.

   Calculate (i) the time to p
   (ii) $|op|$, in surd form.

   Later, the same particle with the same initial speed and final range, was projected at a different angle, θ.

   (iii) Calculate $|op|$, in terms of θ.
   (iv) Find the value of θ, $0° \leqslant \theta \leqslant 90°$.

4. Particles of mass 20 kg and $10\sqrt{3}$ kg, are connected by a light inelastic string passing over a pulley. The masses are held at rest on the smooth plane faces of a triangular wedge and then released.



   (i) Show, in separate diagrams, the forces acting on each particle during the motion.
   (ii) Find, in surd form, the acceleration of the system.
   (iii) If the system is brought to rest and a mass m added to one of the particles causing the system to remain at rest, calculate m.

5. A uniform ladder of weight, W, and length $2l$, rests in limiting equilibrium, the top of the ladder against a rough vertical wall and its base on rough horizontal ground. The coefficient of friction at the wall is $\frac{1}{2}$ and at the ground is $\frac{1}{3}$.

   (i) In a diagram show all forces acting on the ladder.
   (ii) Calculate the normal reactions in terms of W.
   (iii) Calculate the measure of the angle between the ladder and the wall.

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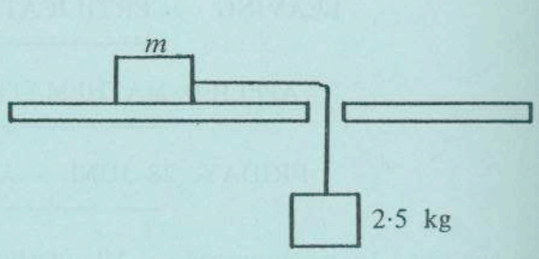
6. A particle of mass m is connected by a light inextensible string which passes through a hole in a table to another particle of mass 2·5 kg which hangs freely and remains at rest. The first particle describes a circle of radius 100 mm on the smooth horizontal table at a constant angular velocity of 2·5 rad/s, the hole being the centre of the circle.



(i) Draw separate diagrams showing the forces acting on each mass.

Calculate, taking $g = 10$ m/s^2

(ii) the tension in the string

(iii) the value of m

(iv) the speed of the particle on the table

(v) the centripetal acceleration of the particle on the table.

7. A smooth sphere of mass 4 kg, moving with a speed of 11 m/s collides directly with a smooth sphere of mass 6 kg, moving in the same direction with a speed of 7 m/s. After collision the 6 kg sphere moves with a speed of 10 m/s.

Calculate (i) the speed of the first sphere after collision

(ii) the coefficient of restitution

(iii) the impulse given to the second sphere

(iv) the overall loss of kinetic energy.

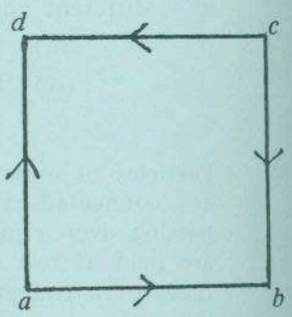
8. A body is in equilibrium under the action of the forces

$$F_1 = 3\vec{i} + 2\vec{j}$$
$$F_2 = x\vec{i} - 5\vec{j}$$
$$F_3 = 2\vec{i} + y\vec{j},$$

where $\vec{i}$, $\vec{j}$ are unit perpendicular vectors.

Calculate the value of x and the value of y.

A square $abcd$ of side 1 m has forces of 10 N, 2 N, 7 N and 6 N acting along the sides ab, cb, cd and ad respectively.



Calculate (i) the magnitude of the resultant force

(ii) the moment of the forces about d.

(iii) the distance from d to the line of action of the resultant.

9. A pencil, of mass 0·035 kg, and of uniform cross-section, floats vertically in water with $\frac{7}{8}$ of its volume, V, submerged. If the density of water is 1000 kg/m^3 calculate, taking $g = 10$ m/s^2,

(i) the weight of the pencil

(ii) the upthrust on the pencil

(iii) the density of the pencil.

If the pencil is completely submerged by pushing it vertically downward with a force F, calculate

(iv) the new upthrust on the pencil

(v) the magnitude of F.