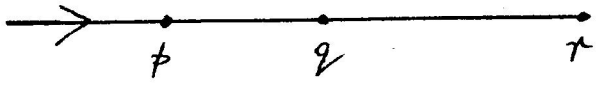
LEAVING CERTIFICATE EXAMINATION, 1982

APPLIED MATHEMATICS - ORDINARY LEVEL

MONDAY, 28 JUNE - AFTERNOON, 2.00 to 4.30

Six questions to be answered.
All questions carry equal marks.
Mathematics tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2.



1. A car travelling towards p at a steady speed of 15 m/s, accelerated at a constant rate between p and q. At q its speed was 25 m/s. This speed was maintained as far as r (see diagram).

If $|pr| = 980$ m and the time from p to r was 40 seconds, draw a time/velocity graph of the motion and, hence, or otherwise, calculate the acceleration.

2. Two planes are flying on straight courses at the same height above level ground. Plane A is travelling at a speed of 500 km/h. Plane B is travelling due north at 400 km/h. At 15.00 hours A is 60 km due east of B and is on a course which will result in collision with B.

(i) Draw a diagram showing the relative positions of A and B at 15.00 hours.

(ii) What is the direction of the velocity of A relative to B ?

(iii) Determine the direction in which A is travelling.

(iv) Calculate the magnitude of A's velocity relative to B.

(v) At what time will the collision occur ?

3. Define momentum. Define a newton. State Newton's second law of motion.

A lift, of mass 1440 kg and carrying 8 people each of mass 70 kg, is being hoisted vertically upwards by a rope in which the tension is 24 kN. Calculate the acceleration of the lift. Determine the reaction between the floor of the lift and *each* person.

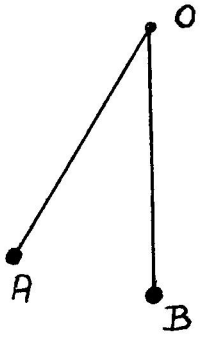
4. Explain the meaning of the word power.

The maximum power output from a train of mass 200 tonnes is 500 kW. Find the maximum acceleration of the train on a level straight track at a speed of 10 m/s if the resistance to motion at this speed is 2 kN.

[1 kW = 1000 W; 1 kN = 1000 N]

Find the maximum acceleration of the same train on an incline of 1 in 196 at a speed of 20 m/s if the resistance to motion at this speed is 9 kN.

5. Two pieces of string, each of the same length, are tied to a fixed point 'o'. The loose end of each string is attached to a particle of mass m. Initially the strings are taut and lie in the same vertical plane. The particle, labelled B in the diagram, is vertically below o. Particle A is released from rest at some height above B and some distance away from B. Just before impact with B, A is moving at 4 m/s. Given that the coefficient of restitution for the collision is $\frac{1}{2}$, find the velocity of A and the velocity of B immediately after the collision. Calculate the height to which B rises in the subsequent motion.



6. (a) An inextensible string 0·5 m in length was fixed at o and had a particle of mass 2 kg attached to its free end (see diagram).

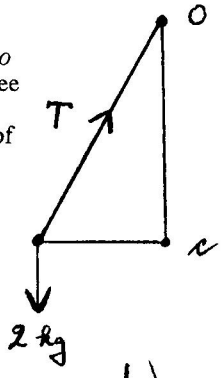
The particle described a horizontal circle, centre c, of radius 0·3 m with uniform speed v m/s. Write expressions for

the acceleration towards c

the centripetal force

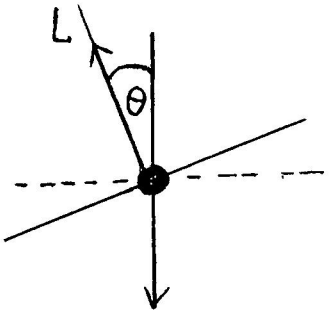
the component of the tension T (i) along (ii) perpendicular to, the radius.

Calculate the value of T and the value of v.



(b) An aircraft of mass 50 tonnes described a horizontal and circular path of radius 4800 m. The sine of θ, the angle of bank, is $\frac{5}{13}$. If L represents the lifting force, calculate the speed of the plane.

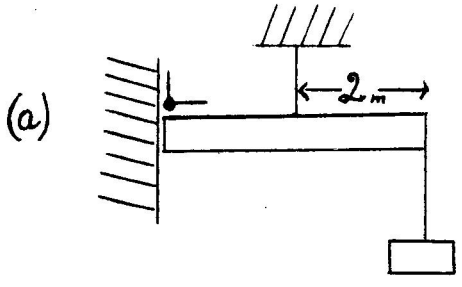
[Take $g = 10$ m/sec^2].

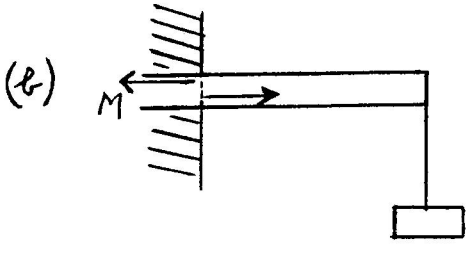


7. Show that the moment of a couple is the same about all points in its plane.

Diagram (a) below shows a uniform beam freely hinged at one end to a vertical wall and supported in a horizontal position by a vertical cable. The beam, whose weight is 2000 N and length 4 m, carries a load of 500 N at its free end. Find the magnitude and direction of the reaction at the hinge and the tension in the cable.

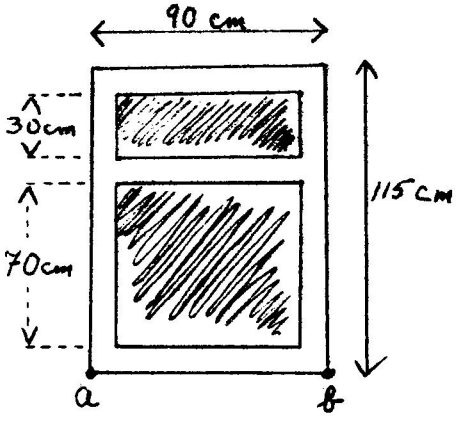
Diagram (b) shows a beam with one end embedded in a vertical wall and carrying a load of 800 N at the other end. The action of the embedded portion of the beam on the exposed portion is equivalent to (i) a supporting force R at the wall and (ii) a couple of moment M. If the exposed portion weighs 3000 N and has a length of 6 m, find the values of R and M.





8. Find the distance, from the bottom edge $[ab]$, of the centre of gravity of a uniform rectangular sheet of plywood 90 cm by 115 cm (see diagram).

The shaded sections were cut out to leave a frame 5 cm wide surrounding the cut out areas. Calculate the distance of the centre of gravity of the frame from the bottom edge $[ab]$ correct to one place of decimals.



9. A solid piece of steel weighs 40 N in air and 35 N in water. Find the relative density of the steel.

An open rectangular tank is constructed from 5 sheets of steel: one sheet for the base - 80 cm by 60 cm, two sheets for the front and back - each 80 cm by 40 cm and two sheets for the ends - each 60 cm by 40 cm. The thickness of each sheet is ·15 cm and the density of the steel is 8 g/cm^3. Calculate the total area of sheet metal used and hence the mass of the tank. If the tank is allowed to float in a lake find the depth of the base below the surface of the lake. Take the density of the lake water to be 1 g/cm^3 and assume the external and internal dimensions of the tank to be the same.