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LEAVING CERTIFICATE EXAMINATION, 1980

APPLIED MATHEMATICS - ORDINARY LEVEL

TUESDAY, 24 JUNE - AFTERNOON, 2.00 to 4.30

Six questions to be answered.
All questions carry equal marks.
Mathematics tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2.

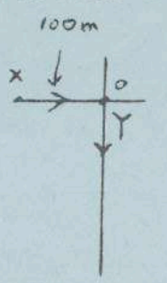
$\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. p and q are points 162 m apart. A body leaves p with initial speed 5 m/s and travels towards q with uniform acceleration 3 m/s^2. At the same instant another body leaves q and travels towards p with initial speed 7 m/s and uniform acceleration 2 m/s^2.

After how many seconds do they meet and what is then the speed of each body ?

2. Particles A and B have velocities $a\vec{i}$ and $b\vec{j}$, respectively. Express the velocity of A relative to B in terms of a, b, $\vec{i}$ and $\vec{j}$ and the magnitude of this velocity in terms of a and b.

Two straight roads cross at right angles at o. A person X is walking along one of the roads towards o at 7 km/hr. Another person Y is cycling along the other road at 24 km/hr. When X is 100 m from o, Y is at o. Find the vector which represents the velocity of Y relative to X and hence calculate the shortest distance between X and Y.



3. Complete the statement: "if a body is in equilibrium under the action of three non-parallel forces, the lines of action of these forces . . . . . "

A uniform rod [ab] 2·5 m in length weighs 240 N. It is freely hinged to a vertical wall at a. A horizontal force P acting at b perpendicular to the wall holds the rod so that b is 1·5 m from the wall.

Draw a diagram showing the lines of action and the direction of all the forces acting on the rod.

Calculate the magnitude of

(i) the force P
(ii) the resultant force at a correct to the nearest newton.

4. Explain the terms (i) limiting friction
(ii) coefficient of friction.

A rough plane is inclined at an angle α to the horizontal. Tan $\alpha = 2$. A body of weight W on the plane is just prevented from slipping down along the plane by a horizontal force W.

Calculate the coefficient of friction between the body and the plane.

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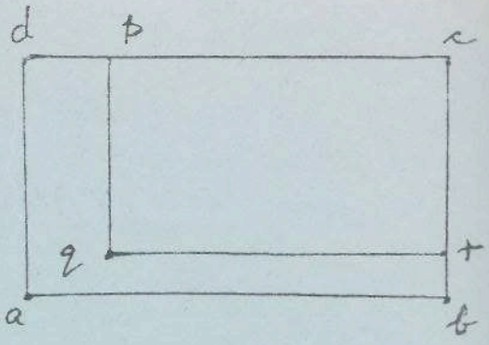
5. $abcd$ is a uniform rectangular lamina and $cpqr$ is a rectangular portion of it (see diagram).



Taking a as origin and $\vec{i}$ and $\vec{j}$ as the unit vectors along ab and ad, respectively, the position vectors of the centres of gravity of $abcd$ and $cpqr$ are respectively

$$(5\vec{i} + 3\vec{j}) \quad \text{and} \quad (6\vec{i} + 3{\cdot}5\vec{j}).$$

Calculate the position vector of the centre of gravity of the portion $abrqpd$.

6. A projectile is fired from a point p on horizontal ground with an initial velocity whose horizontal and vertical components are 17·5 m/s and 21 m/s, respectively. Calculate the height of the projectile above the ground two seconds after being fired.

t seconds later the projectile is again at the same height. Calculate t and find the horizontal displacement of the projectile $(t + 2)$ seconds after being fired.

7. A sphere A of mass 2 kg moving at 10 m/s collides directly with a stationary sphere B of mass 3 kg. The coefficient of restitution between the spheres is $\frac{1}{2}$.

Calculate
(i) the speed of each sphere after the collision
(ii) the loss in kinetic energy due to the collision
(iii) the power exerted by an agent which brings sphere B to rest again after 2 seconds.

8. A particle P of mass 4 kg is on a smooth horizontal table. It is connected by a light, inextensible string which passes through a small hole in the table to a particle Q of mass 2·5 kg.

P describes a circle of radius 0·5 m on the table, the centre of the circle being at the hole, and as a result Q is kept hanging freely underneath the table at a height above floor level greater than 0·5 m.

Find the angular velocity of P.
If P is stopped, held and then released, calculate the acceleration of P and show that its speed on reaching the hole is $\frac{7}{\sqrt{13}}$.

9. A large cylindrical vat has internal diameter of length 2·8 m. It contains liquid of relative density 1·25 to a depth of 2 m. If $\pi = \frac{22}{7}$, calculate in kilonewtons, the thrust due to the liquid on the horizontal base of the vat.

If a solid body of mass 750 kg and relative density 2·5 is lowered by means of a fine wire into the liquid until it hangs fully immersed what, in kilonewtons, is the consequent increase in the thrust on the base ?
(Density of water = 1,000 kg/m^3.)