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LEAVING CERTIFICATE EXAMINATION, 1979

APPLIED MATHEMATICS–ORDINARY LEVEL

FRIDAY, 22 JUNE - AFTERNOON, 2 to 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2.
$\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. A train travels between two stations A and B which are 5 km apart. It starts from rest at A and is uniformly accelerated for 15 seconds attaining a speed of 20 m/s. This speed is maintained until the train is 250 m from B, when it is decelerated uniformly to rest at B. Calculate

(i) the acceleration,
(ii) the deceleration,
(iii) the total time taken in travelling from A to B.

2. A uniform ladder lies with one end in contact with a rough vertical wall where the coefficient of friction is $\frac{1}{2}$. The base of the ladder lies on rough horizontal ground where the coefficient of friction is $\frac{1}{3}$. The ladder makes an acute angle θ with the horizontal.

If the ladder is on the point of slipping, show that $\tan \theta = \frac{5}{4}$.

3. A vertical tower 40 m high stands on level ground. A stone is projected from the top of the tower with an initial speed of 35 m/s and at an angle of elevation α, where $\tan \alpha = \frac{3}{4}$. Calculate

(i) the maximum height it reaches above the ground,
(ii) the time taken by the stone to reach the ground,
(iii) the horizontal distance travelled by the stone during its flight.

4. abc is an isosceles triangle with $|ab| = |ac| = 10$ cm and $|bc| = 16$ cm. Find the distance of the centre of gravity from the side bc

(i) if the triangle is formed from a 36 cm length of uniform wire,
(ii) if the triangle is a uniform lamina.

If the triangle is a uniform lamina and the portion axy is removed, where x and y are the mid-points of the sides ab and ac respectively, find the centre of gravity of the remainder.

5. Two elastic spheres of mass 3 kg and 2 kg collide directly when moving in the same straight line and in the same direction. The speeds before collision are 7 m/s and 3 m/s respectively. If the coefficient of restitution is $\frac{1}{4}$, calculate the velocities after the collision.

Find also the loss in kinetic energy due to the collision.

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6. State the principle of moments.

What is the algebraic sum of the moments of a system of coplanar forces about a point on their resultant ? Explain your answer.

abcd is a square. Forces of 4N, 7N, 1N and 5N act in the directions $\vec{ab}$, $\vec{cb}$, $\vec{dc}$, $\vec{da}$, respectively. Find the magnitude of the resultant. Prove that its line of action divides *ab* in the ratio 2 : 1.

7. A mass of 5 kg lies on a smooth horizontal plane. It is connected, by means of a light inextensible string passing over a smooth light pulley at the edge of the plane, to a mass of 2 kg hanging freely. Find the common acceleration when the system is released from rest.

How far do the masses travel in the first half-second ? If the string were then cut, how far would each of the masses travel in the next half-second ?

8. State the principle of conservation of energy.

A particle hangs freely from a fixed point O by means of a light inextensible string of length 0·2 m. The particle is pulled aside until the string makes an angle of 60° with the vertical and is then released from rest. Find the velocity of the particle when it reaches a point vertically below O.

Calculate the angle the string makes with the vertical when the particle has a speed of 0·7 m/s, correct to the nearest degree.

9. (a) A rectangular tank 2 m long and 1·5 m wide is filled to a depth of 0·5 m with a liquid of relative density 0·8. Calculate, in newtons, the thrust exerted by the liquid on the horizontal base of the tank.
(Density of water = 1000 kg/m^3)

(b) A piece of metal of mass 0·375 kg floats freely in mercury with $\frac{4}{9}$ of its volume above the surface of the mercury. If the relative density of mercury is 13·5, calculate the relative density and the volume of the metal.