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LEAVING CERTIFICATE EXAMINATION, 1978

APPLIED MATHEMATICS – ORDINARY LEVEL

WEDNESDAY, 21 JUNE – AFTERNOON, 2 to 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2
$\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. A particle starting from rest moves in a straight line with an acceleration of $\frac{1}{2}$ m/s^2 for 8 seconds. It then decelerates uniformly to a speed of 3 m/s in the next 4 seconds. It maintains this speed for 5 seconds and then comes to rest with a deceleration of 2 m/s^2. Draw a velocity–time graph for this motion.

Hence, or otherwise, find:-

(i) the greatest velocity reached during the journey,
(ii) the total time for the journey,
(iii) the total distance travelled.

2. A particle P is moving with velocity $-9\vec{i} + 25\vec{j}$ while another particle Q is moving with velocity $6\vec{i} + 10\vec{j}$. Find the velocity of P relative to Q.

P is at the point $10\vec{i}$ when Q is at the origin O. Show those positions of P and Q on a diagram and show the path of P relative to Q.

Calculate the distance of O from this path. What does this distance represent ?

3. A uniform rod of weight $6W$ is freely hinged at one end to a fixed point. The rod is pulled aside by a horizontal force F applied at its other end, so that it makes an angle 30° with the vertical. Find F in terms of W. What angle would the rod make with the vertical if the horizontal force was $3W$?

4. A particle is projected from a point P with initial velocity $10\vec{i} + 15\vec{j}$ m/s, where $\vec{i}$ is measured along the horizontal and $\vec{j}$ along the vertical. A vertical wall 5 metres high stands 20 metres from P on the same horizontal level. Show that the particle will pass over the wall.

On which part of the flight, ascent or descent, will this occur ?

5. A mass of 5 kg is placed on a rough plane inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$. A string attached to the 5 kg mass passes over a smooth pulley at the top of plane and supports a 2 kg mass hanging freely at its other end. The string is parallel to the line of greatest slope. If the 5 kg mass is on the point of slipping down the plane, calculate the coefficient of friction.

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6. Four points p, q, r, s are determined by the vectors $2\vec{i} - 4\vec{j}$, $4\vec{i} + 3\vec{j}$, $-2\vec{i}$, and $-\vec{i} + 2\vec{j}$, respectively. Particles of weight 5N, 2N, 1N, and 4N are placed at p, q, r, s respectively.

Express the centre of gravity of the system in terms of $\vec{i}$ and $\vec{j}$. Where should an extra particle of weight 6N be placed so that the centre of gravity of the system would be at $2\vec{i} + 3\vec{j}$?

7. Two elastic bodies, of mass, 1 kg and 2 kg, travelling in opposite directions collide directly. The speeds before collision are 16 m/s and 9 m/s, respectively. If the coefficient of restitution is $5/7$, calculate the velocities after the collision.

Show that kinetic energy is lost in the collision.

8. A particle of mass 3 kg is connected by means of a light string of length 0·6 m to a fixed peg O. The particle is describing a horizontal circle with centre directly below O at an angular speed of 5 radians per second. Find the tension in the string.

Calculate, to the nearest degree, the angle the string makes with the vertical.

9. State the Principle of Archimedes.

A piece of wood in the shape of a right circular cylinder of height 0·21 m floats with its axis vertical in a tank of liquid of relative density 1·05. If the relative density of the wood is 0·80 what length of the cylinder is under the liquid ?

The cylinder is then held totally submerged by means of a light string attached to the midpoint of the base of the cylinder and to a point on the bottom of the tank. Given that the area of the base of the cylinder is 0·04 m^2 calculate, to the nearest Newton, the tension in the string.
(Density of water = 1000 kg/m^3).