AN ROINN OIDEACHAIS M.51

LEAVING CERTIFICATE EXAMINATION, 1977

APPLIED MATHEMATICS - ORDINARY LEVEL

FRIDAY, 24 JUNE - Morning, 9.30 to 12

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2.
$\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. Derive the formula $v^2 - u^2 = 2as$ for linear motion under constant acceleration a, where u and v are the initial and final velocities, respectively, and s is the distance travelled.

A particle is moving in a straight line. It passes through a point P at a speed of 2 m/s and moves with constant acceleration 6 m/s^2 until its speed is 10 m/s. The direction of the acceleration is then reversed, its magnitude being the same (6 m/s^2). Find how far from P the particle has travelled when its speed becomes zero, and find the speed at which it passes through P again.

2. Explain, with the aid of a diagram, what is meant by the relative velocity of one body in respect of another.

Two particles A, B have velocities $7\vec{i} - 4\vec{j}$ and $3\vec{i} - \vec{j}$, respectively. Find the velocity of A relative to B.

Show that the particles will collide if the position vector of A is $6\vec{j}$ when the position vector of B is $8\vec{i}$.

3. A uniform ladder has one end in contact with a smooth vertical wall while the other end lies on rough horizontal ground. The coefficient of friction is $\frac{1}{2}$. Calculate the angle that the ladder makes with the horizontal when it is on the point of slipping.

4. When a block is put on a rough plane inclined at an angle α to the horizontal, where $\tan\alpha = 3$, a horizontal force equal in magnitude to twice the weight of the block is just sufficient to keep it from sliding down the plane. Find the coefficient of friction.

5. A particle is projected from a point which is 30 m horizontally from the foot of a vertical tower. After 3 seconds it strikes the tower at a height of 27·9 m. Find the initial speed of the particle and the angle of projection.

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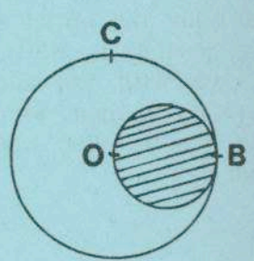
6. Write down an expression for the resultant force F acting on an object of mass m moving in a circle of radius r with constant speed v.

A particle is moving with constant speed 7 m/s in a horizontal circle of radius r metres along the smooth inside surface of an upturned cone of semivertical angle α. Show in a diagram the forces acting on the particle and indicate the resultant force. Find r in terms of α, and hence show that the plane of the circle is 5 m above the apex of the cone.

7. Two masses, of 5 kg and 10 kg respectively, are suspended from the ends of a light inextensible string passing over a smooth pulley, and are released. Find the speed of the masses when the smaller one has risen a distance of 0·3 m.

If the string is then cut, how much further will the smaller mass rise ?

8. A circular piece is cut out of a uniform circular disc of centre O. The diameter of the piece is the radius OB of the disc (see diagram). Find the centre of gravity of the remainder.



The remainder of the disc is then suspended from a point C on its circumference, where OC ⊥ OB. Find the angle which CO makes with the vertical.

If the mass of the piece cut out is m, what mass attached to the remainder at B would make CO vertical ?

9. Two spheres, of masses 2 kg and 4 kg, collide directly when moving in opposite directions with speeds of 15 m/s and 10 m/s, respectively. If the heavier sphere is brought to rest by the collision, find the velocity of the other sphere after impact.

Calculate the loss in kinetic energy due to the collision, and calculate the coefficient of restitution between the spheres.

10. Define simple harmonic motion. Show that the motion given by $x = A \sin \omega t$, where A and ω are constants, is simple harmonic motion.

A particle is moving with simple harmonic motion. When it is 4 m from the midpoint of its path its acceleration is 25 m/s^2 and its velocity is 7·5 m/s. Find the periodic time and the amplitude.