4280

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LEAVING CERTIFICATE EXAMINATION, 1976

APPLIED MATHEMATICS – ORDINARY LEVEL

FRIDAY, 25 JUNE – Morning, 9.30 to 12

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be $9{\cdot}8$ metres/second^2.
$\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

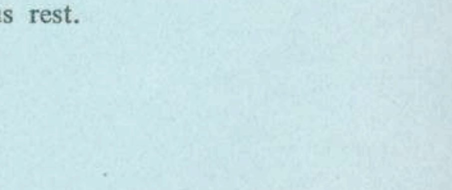
1. A bus accelerates uniformly for 10 s as it goes from rest to its maximum speed v m/s, while travelling 100 m on a straight road. Find its acceleration and the value of v. The bus continues at its maximum speed for two minutes and then decelerates uniformly to rest in 5 s. Find the total distance covered during the journey.

2. Show in a vector diagram how the velocity of A relative to B is related to the velocity of A and the velocity of B.
A ship is travelling in a direction due North at 6 m/s. A passenger on board walks across the deck at right angles to the direction of motion of the ship at a speed of $2{\cdot}5$ m/s relative to the ship. Find the actual velocity of the passenger.

3. State the principle of the conservation of linear momentum.
A car and a lorry are travelling along roads *ao* and *bo* which intersect at right angles. The car, of mass 400 kg and travelling at 30 m/s, crashes into the lorry which is of mass 1200 kg and travelling at 20 m/s. Assuming that the vehicles coalesce on impact, calculate the components of the velocity of the combination immediately afterwards.

4. State the principle of the conservation of energy for a mechanical system.
A particle hangs freely at rest at the end of a light inelastic string of length $2{\cdot}5$ m which is tied to a fixed point o. The particle is then projected horizontally with a speed of 7 m/s. Find
   (i) the speed of the particle when the string is inclined at $60°$ to the vertical,
   (ii) the position of the particle when it comes to instantaneous rest.

5. State Newton's second law of motion.
The particle A of mass 3M rests on a smooth horizontal table (see diagram). It is connected by two light inelastic strings, passing over smooth pulleys at opposite edges of the table, to two particles B and C of masses M and 2M, respectively, hanging freely. The system starts from rest with the strings taut. Show in separate diagrams the forces acting on the three particles and calculate
   (a) the common acceleration of the system,
   (b) the tensions in the strings.



6. A particle of mass 1 kg is attached to one end of a light inelastic string of length 0·5 m and the other end of the string is fixed at a point *o*, which is 0·3 m above a smooth horizontal table. The particle is sliding on the table moving in a circle, whose radius is 0·4 m and whose centre *c* is vertically below *o*, with angular speed 5 radians per second. Show in a diagram the forces acting on the particle during the motion. Find the tension in the string and the reaction of the table on the particle.

7. A stone is projected horizontally with speed 28 m/s from the top of a vertical tower, which is 90 m above level ground. Find the time taken for the stone to reach the ground and find how far the point of contact is from the foot of the tower.

8. (i) Prove that the moment of a couple about a point is the same for all points in its plane.

(ii) Four forces of magnitudes 10, 20, 30, 40 newtons act along the sides of a square *abcd* - lettered anticlockwise - of side 1 m, in the directions of $\vec{ab}$, $\vec{bc}$, $\vec{cd}$, $\vec{da}$ respectively. There is also a couple of anticlockwise moment 30 Nm acting in the plane of the square. Find the magnitude and direction of the resultant of this system, and show that it passes through *d*.

9. Prove that the lines of action of three non-parallel coplanar forces in equilibrium are concurrent.

A uniform rod *ab* of weight 100 N and length 2 m is suspended by two light inelastic strings *ao* and *ob*, each of length 2 m, from a fixed point *o*. Show in a diagram the forces acting on the rod. Calculate the tensions in the strings.

10. Prove that the difference of the pressures at two points in a fluid is proportional to the difference of their depths.

A cylinder is 2 m long and is filled with liquid of specific weight 1000 N/m^3, with the sides vertical and the base horizontal. Two small windows each of area 0·01 m^2 are in the cylinder, one half way down the side of the cylinder and the other in the base. Assuming the pressure is the same at all points on the window in the side of the cylinder, calculate the forces exerted by the liquid on the two windows.