AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1975

APPLIED MATHEMATICS – ORDINARY LEVEL

THURSDAY, 26 JUNE – Morning, 9.30 to 12

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be $9 \cdot 8$ metres/second^2.
$\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. A particle is projected vertically upwards under gravity with a speed of 14 m/s. Find the greatest height attained by the particle above the point of projection and the time taken to reach this height.

2. Explain what is meant by the relative velocity of a moving point p with respect to a moving point q.
A train is travelling on a straight track with velocity $25\vec{j}$ and a car visible from the train is travelling on a straight road with velocity $20\vec{i} + 10\vec{j}$, where speeds are measured in m/s. Calculate the magnitude and direction of the car's velocity as it appears to a person sitting in the train.

3. Explain what is meant by the principle of the conservation of mechanical energy.
A particle of mass 5 kg slides from rest down the line of greatest slope on a smooth plane inclined at 30° to the horizontal. If the particle travels 20 m down the plane, find the change in its potential energy and show that the particle attains a speed of 14 m/s.

4. Prove that a force $\vec{F}$ and a couple M in the same plane are equivalent to a single force.
A force of 50 N acts along the side [ab] of an equilateral triangle abc, lettered anticlockwise, of side 1 m in length, and there is a couple of clockwise moment $25\sqrt{3}$ Nm acting in the plane of the triangle. Find the resultant of this system of forces.

5. A particle of mass 2M is held on a rough horizontal table where the coefficient of friction is $\frac{1}{3}$ and is connected by a light inelastic string passing over the smooth edge of the table to a particle of mass M hanging freely under gravity. Show in separate diagrams the forces acting on the two particles when the system is released from rest. Calculate the common acceleration of the particles.

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6. Four particles of the following weights are placed at four points a, b, c, d in a plane with the following position vectors relative to a fixed origin o

$$2\text{ N at } a(4\vec{i} + 3\vec{j}); \quad 3\text{ N at } b(2\vec{i} - 3\vec{j}); \quad 4\text{ N at } c(-5\vec{i} + 6\vec{j}) \text{ and } 5\text{ N at } d(4\vec{i} - 7\vec{j}).$$

Find the position vector of the centre of gravity of the system.

Where should a fifth particle of weight 7 N be placed in order that the centre of gravity of the five particles be at o ?

7. A light inextensible string of length 0·4 m has one end p fixed and carries a particle of mass 5 kg at the other end q. The particle describes a horizontal circle, whose centre is vertically below p, at a uniform angular speed of 7 rad/s. Show in a diagram the forces acting on the particle and write down two equations involving the tension T in the string and its inclination θ to the vertical. Calculate T and θ.

8. A light inextensible string is tied to two points a, b, on the same horizontal level and a ring of weight 70 N can slide smoothly on the string. When hanging under gravity, the ring is pulled aside by a horizontal force F acting parallel to ab.

In the position of equilibrium the two parts of the string are inclined to the horizontal at angles of α and $90 - \alpha$, where $\sin\alpha = \frac{3}{5}$. Show in a diagram the forces acting on the ring. Calculate the tension in the string and find the value of F.

9. Three billiard balls a, b, c of equal mass are moving, in that order and not in contact, in a straight line on a smooth horizontal table. The velocities of a, b, c are $8\vec{i}$, $4\vec{i}$, and $u\vec{i}$, respectively. If a first collides with b and the coefficient of restitution is $\frac{1}{2}$, calculate the velocities of a and b after the collision. Find the least value of u so that there are no further collisions.

10. A solid cube, with edges of length l, is held immersed in a fluid of specific weight w with its upper plane face horizontal and at a depth $2l$ below the surface. Calculate the forces exerted by the fluid on the upper and lower faces. Verify that these values are in agreement with Archimedes principle.