AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1974

APPLIED MATHEMATICS – ORDINARY LEVEL

(400 marks)

FRIDAY, 28 JUNE – Morning, 9.30 to 12

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2.
$\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. Two engines A and B travelling on parallel rails in the same direction with uniform accelerations of 4 m/s^2 and 6 m/s^2, respectively, simultaneously pass a signal box P with speeds of 20 m/s and 10 m/s, respectively. Write down the distance from P travelled by each engine in t seconds. Find when and where the engines are level again. Draw on the same graph paper a velocity–time graph for the motion of each engine. From the graph estimate how many seconds after passing P are the two speeds the same.

2. Show in a diagram how to find the true velocity of a boat given the velocity of the river and the velocity of the boat relative to the river.

A boat crosses a river, 100 m wide, which flows parallel to the straight banks with velocity $\vec{v}_1 = 4\vec{i}$ while the velocity of the boat relative to the river is $\vec{v}_2 = 3\vec{j}$, where v_1, v_2 are measured in m/s. Find the magnitude of the true velocity of the boat. Find how long it takes the boat to cross the river and the distance it has travelled in doing so.

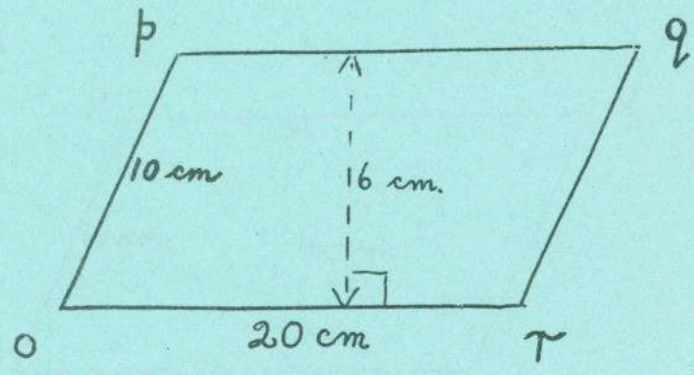
3. A particle of mass 2 kg moving with a speed of 10 m/s collides directly with a stationary particle of mass 1 kg. The collision is a perfectly elastic one ($e = 1$). Write down two equations involving the speeds of the particles after the collision. Show that there is no loss in the kinetic energy of the system.

4. What is meant by the Conservation of Linear Momentum ?

A bullet of mass 0·1 kg moving horizontally with velocity $200\vec{i}$ strikes and becomes imbedded in a block of wood of mass 10 kg. Before the collision the block moves horizontally with velocity $10\vec{j}$, where the speeds are measured in m/s. Find the velocity of the compound body after the collision.

5. The position vectors, relative to a fixed origin, of two particles of weights w_1 and w_2 are $a_1\vec{i} + a_2\vec{j}$ and $b_1\vec{i} + b_2\vec{j}$, respectively. Find an expression for the position vector of their centre of gravity.

opqr is a parallelogram, with perpendicular height 6 cm, as in diagram. o is the fixed origin and $\vec{i}$, $\vec{j}$ the unit vectors along or and perpendicular to or, respectively. Four particles of weights 13, 5, 8, 4 N are placed at o, p, q, r, respectively. Find in terms of $\vec{i}$ and $\vec{j}$ the position vectors of p, q and r and hence the position vector of the centre of gravity of the four particles.



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6. State the conditions satisfied by a set of concurrent forces if they are in equilibrium.

A particle of mass 10 kg is acted on by the forces $\vec{F}_1 = 2\vec{i} - 3\vec{j}$, $\vec{F}_2 = 5\vec{i} + 5\vec{j}$, $\vec{F}_3 = 5\vec{i} + 14\vec{j}$, where the forces are measured in newtons. Calculate the magnitude of the acceleration that such a system would produce on the particle.

Find the additional force, $\vec{F}_4$, that would reduce the system to equilibrium.

7. Prove that a particle moving in a circle of radius b with constant angular speed ω is accelerated towards the centre by an amount $b\omega^2$.

The moon, of mass m_1 kg, is attracted to the earth, of mass m_2 kg, by a force of magnitude $\frac{Gm_1m_2}{r^2}$, where r metres is the distance between them and G is the gravitational constant $6{\cdot}66 \times 10^{-11}$ $m^3/kg\,s^2$. Assuming that the moon completes a circular orbit of radius 384000 km about the earth as centre in 27 days, calculate m_2 approximately and give your answer in the form $n \times 10^{24}$ kg, where $n \in N$.

8. A particle is projected with velocity 50 km/s at an angle θ to the horizontal. If $\vec{i}$ is along the horizontal and $\vec{j}$ is vertically upwards and if $\cos\theta = \frac{3}{5}$, express the position vector $\vec{r}$ of the particle after t seconds of motion in terms of $\vec{i}$ and $\vec{j}$.

Find the magnitude to the nearest metre and the direction to the nearest degree of the velocity of the particle when $t = 1$.

Calculate the horizontal range of the particle to the nearest metre.

9. Two particles each of mass m are connected by a taut inelastic string as they slide across a horizontal table with common speed u. The contact between the leading particle and the table is perfectly smooth while the second particle is rough and the coefficient of friction between it and the table is $\frac{1}{2}$. Show in separate diagrams the horizontal forces acting on each particle. Calculate the tension in the string and the distance travelled by the particles in coming to rest.

10. Prove that the pressure at a point in a homogeneous liquid is proportional to its depth below the surface.

A hollow right circular conical vessel of height h and base area a is filled with liquid of weight w per unit volume. The vessel is held with its axis vertical and its vertex above the base. Calculate the downward force exerted on the base. Deduce that the force exerted by the curved surface of the vessel on the liquid is twice the weight of the liquid.