AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION. 1972

APPLIED MATHEMATICS - ORDINARY LEVEL

(400 marks)

FRIDAY, 23rd JUNE - Morning, 9.30 to 12

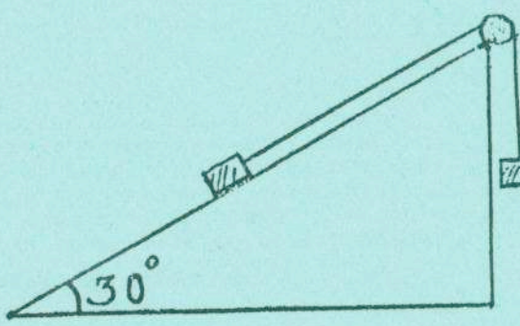
Not more than six questions may be answered.
All questions are of equal value.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2.

1. A car starts from rest with a constant acceleration of 5 m/s^2. Find the time taken and the distance travelled until it reaches its maximum speed of 30 m/s. On reaching this speed it immediately decelerates uniformly to rest in a further 3 s. Find the total distance travelled and draw a rough velocity-time graph of the motion.

2. A uniform triangular lamina abc is right angled at b, with $|ab| = 0{\cdot}5$ m, $|bc| = 1{\cdot}2$ m. The points p and q are the midpoints of the sides $[bc]$ and $[ac]$, respectively. If the triangular section cqp is now cut away, find the distances of the centre of gravity of the remainder $abpq$ from the sides $[ab]$ and $[bp]$.

3. A sphere of mass 5 kg moving on a smooth horizontal plane at 2 m/s collides directly with a similar sphere of mass 6 kg moving in the opposite direction at 4 m/s. If the coefficient of restitution between the spheres is 0·1, find their speeds after impact. Prove that the magnitude of the impulse exerted by either sphere on the other during collision is 18 Ns.

4. A particle of mass 20 kg is held at rest on a smooth plane inclined at 30° to the horizontal. A light inelastic string connects this particle, over a smooth pulley at the top of the plane, to a second particle of mass 15 kg hanging freely. The part of the string on the plane lies along a line of greatest slope and the other part is vertical. Show in separate diagrams the forces acting on each particle when they are released from rest. Calculate their common acceleration and the tension in the string in the subsequent motion. (See diagram).



5. A uniform ladder of weight 200 N and length 2·5 m rests in equilibrium with one end, a, on a smooth horizontal floor and the end, b, against a smooth vertical wall. The end, a, is attached by a light string of length 0·7 m to the junction of the wall and the floor. Show in a diagram the four forces acting on the ladder. Prove that the tension in the string is $\frac{175}{6}$ N and find the reactions at the wall and at the floor.

6. Define the relative velocity of A with respect to B in terms of the velocity of each. A plane flies from a point p to a point q, distance 600 km due North of p. The pilot flies the plane with a velocity $(5\vec{i} + 110\vec{j})$ m/s relative to the wind which is blowing with a velocity $(-5\vec{i} + 10\vec{j})$ m/s, where $\vec{i}$ and $\vec{j}$ are unit vectors pointing East and North, respectively. Find the actual velocity of the plane and the time taken for the journey. If $(5^2 + 110^2)^{\frac{1}{2}}$ m/s is the maximum speed of the plane in still air, find the time taken for the return journey.

7. A projectile is fired with an initial velocity of $(10\vec{i} + 49\vec{j})$ m/s from a point p, where $\vec{i}$, $\vec{j}$ are unit vectors along the horizontal and upward vertical, respectively. Write down the velocity $\vec{v}$ and the displacement $\vec{r}$ from p of the particle after time t seconds. Find the values of t and x when the displacement has the form $\vec{r} = (x\vec{i} + 122{\cdot}5\vec{j})$ m.

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8. What is the principle of the conservation of energy of a mechanical system ?
Two particles of masses 2 M and M, respectively, are attached to the ends a and b of a light rod ab of length 2 l, which is free to rotate in a vertical plane about a smooth horizontal axis through the midpoint of the rod. Initially the rod is vertical with the 2 M mass uppermost and it is slightly displaced from rest. Find the speed of the particles when the rod is first inclined at 60^{o} to the vertical.

9. A particle of mass 15 kg describes a horizontal circle with constant speed v, on the smooth inside surface of a fixed sphere, of internal radius 0·5 m and centre q. The centre of the circle is 0·3 m below q. In a diagram show the forces acting on the particle, and prove that the reaction of the sphere is 245 N and find the value of v.

10. State Archimedes Principle for a body immersed in a fluid.
A solid cube of side 0·5 m is in equilibrium totally immersed in a vessel of water, being tied to the bottom of the vessel by a light inextensible string. Show in a diagram the forces acting on the cube. If the tension in the string is 490 N, find the density of the material in the cube. (See diagram).
{1 m^3 of water has a mass of 10^3 kg}.