AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1971

APPLIED MATHEMATICS - ORDINARY LEVEL

WEDNESDAY, 23rd JUNE - Morning, 9.30 to 12

Not more than six questions may be answered.
All questions are of equal value.
Mathematical Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2.

1. Show how the formula $s = ut + \frac{1}{2}at^2$, where u and a are constants, represents distance s travelled in a straight line with constant acceleration in time t.

A car travelling along a straight road is accelerating at a constant rate of 1·5 metres per second^2 as it passes a point p with a speed of 12 metres per second. It stops accelerating after 10 seconds and continues moving with constant speed. Find

(i) its speed, and its distance from p, after 8 seconds,

(ii) its distance from p when it stops accelerating,

(iii) its speed after 12 seconds.

2. $abcd$ is a thin uniform rectangular sheet of metal such that ab is 0·6 m and bc is 0·4 m. If a square of edge 0·2 m with one corner at a is cut away, show that the centre of gravity of the remainder is 0·34 m from ad and 0·22 m from ab.

3. A cruiser is sailing due East at 10 metres/second and a destroyer which is 10 kilometres due South of it is sailing at $10\sqrt{2}$ metres/second in a North-Easterly direction. Find the velocity of the destroyer relative to the cruiser. Hence show that the ships will meet after $16\frac{2}{3}$ minutes if they keep sailing with these velocities.

4. A wooden block of mass 4 kilograms, held on a smooth horizontal table and connected by a light inextensible string passing over the smooth edge of the table to a particle of mass 2 kg hanging freely is released from rest. Show in separate diagrams the forces acting on the block and on the particle. Calculate the common acceleration of the block and the particle, and the tension in the string.

5. A bullet is fired horizontally, with a speed of 100 m/s, from the top of a cliff, 1960 metres above sea level. Show that it will strike the sea at a distance of 2000 metres from the foot of the cliff.

6. A uniform rod ab of length $4l$ and weight W, is hinged to a vertical post at a and is supported in a horizontal position by a string of length $5l$ attached at b and tied to a point c, distance $3l$ above a, on the post. Find the horizontal and vertical components of the reaction at the hinge at a, and show that the tension in the string is $5W/6$.

7. State the principle of the Conservation of Linear Momentum in the case of collision between elastic spheres.

A billiard ball of mass 0·5 kg is at rest on a smooth horizontal table when it is struck directly by an equal ball moving along the table with a speed of 2 m/s. If the coefficient of restitution between the balls is $\frac{1}{3}$, find the speed of each ball after the collision.

8. What is the principle of the Conservation of Energy for a particle in motion under conservative forces ?

A small ring is threaded on a smooth piece of wire bent in the form of a circle of radius $1\frac{2}{5}$ metres with its plane vertical. The ring is projected along the wire from the lowest point with a speed of 7 metres/second. Using the conservation of energy show that the speed of the ring, when it is at the same height as the centre of the circle, is $7/\sqrt{3}$ metres/second. Find where the ring comes to instantaneous rest.

9. What is the magnitude and direction of the force needed to make a particle of mass m travel with constant speed v in a circle of radius r ?

A particle of mass 2 kilograms is describing a circle, of radius 0·5 metres, with constant speed v on a smooth horizontal table. A light inextensible string is tied to the particle and passes through a small smooth hole in the table at o, the centre of the circle. To the other end of the string is attached a particle of mass 5 kilograms, which is hanging freely at rest under the table. Find the tension in the string and show that v = 3·5 metres/second.

10. Show that the moment of a couple is the same about any point in its plane.

A rectangle $abcd$ has sides ab = 4 metres and bc = 3 metres. Forces of magnitudes 40, 50, 70, 80, 100 newtons act respectively along the lines da, ba, bc, dc, ca. Find the magnitude of the resultant of these forces and the angle it makes with the side ab.