LEAVING CERTIFICATE EXAMINATION, 1967

APPLIED MATHEMATICS - PASS

THURSDAY, 22nd JUNE - Afternoon, 2.30 to 5

Not more than <u>six</u> questions may be answered.
All questions are of equal value.
Mathematical Tables may be obtained from the Superintendent.

1. A stone is thrown vertically up from the ground with initial velocity 50 ft. per second.

   (a) Calculate its velocity after $1\frac{1}{4}$ seconds.
   (b) How far above the ground will it be after $1\frac{1}{4}$ seconds ?
   (c) What is the highest point (A) to which it will rise ?
   (d) If the stone weighs 2 lbs. what is its energy at A ?

2. Deduce that a particle moving with uniform speed in a circle of radius R, and completing a revolution in time T has velocity (v) of magnitude $\frac{2\pi R}{T}$ and acceleration of magnitude $\frac{v^2}{R}$.

   A particle lies 2 inches from the centre on the turn-table of a gramophone. The turn-table revolves steadily making 45 revolutions per minute. What is the acceleration of the particle ?

3. Explain "work" and "power".
   In 16 seconds a man weighing 11 stone runs from ground level up a stairs to a height of 64 feet above the ground. Calculate in horsepower the rate at which he works against the force of gravity.

4. Write a short note on friction.
   The diagram shows the path of motion of a heavy particle which is released from rest at the point A and under gravity slides inside a container with rough planar sides and bottom, finally reaching the point B. Discuss the magnitude and direction of the forces along AC, CD and DB which govern the motion of the particle.
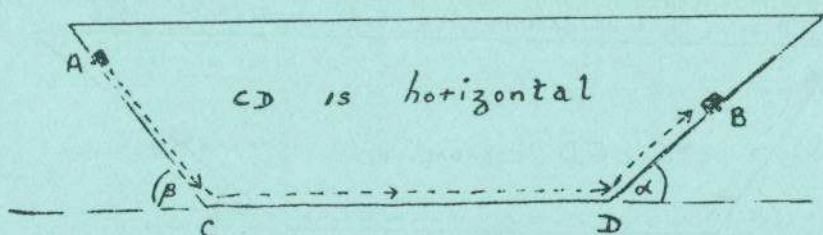
5. A body is placed on a rough inclined plane, and the angle of inclination is slowly increased. Show that the angle of inclination at the instant the body is on the point of sliding down the plane is equal to the angle of friction.
   A block of weight 1 lb. is placed on a rough plane which is inclined at an angle of 60° to the horizontal. A horizontal force P in the same plane as the other forces is applied to the block. If the coefficient of friction for the system is 0·6, calculate the least value of P which prevents the block from sliding down the plane.

6. Given an irregularly shaped piece of sheet cardboard, explain how you would locate its centre of gravity.
   A square sheet of cardboard ABCD weighing 6 ounces is suspended by a light string fastened at the corner A. A weight of 3 ounces is fastened to the adjacent corner B. If the side of the square is 12 inches long calculate the tangent of the angle made by the diagonal BD to the horizontal.

7. A straight river flows uniformly at two m.p.h. An observer on the bank sees two boats moving at maximum velocity. One boat is moving with the current, while the line of motion of the other is perpendicular to the direction of the current. If each boat has a maximum speed of four m.p.h. in still water, calculate the magnitude of the velocity of one relative to the other.

8. Show that the rate of change of the momentum of a particle is a force.
   A body of mass 500 grams falling vertically from rest from the top of a building under a constant force has momentum equal to two kilogram metres per second after one second.

   (i) Show that its fall is not due to gravity alone.
   (ii) Calculate the height of the building if the body strikes the ground after two seconds.

                          (g = 981 cm. per second per second.)

9. Describe how you would show experimentally that the liquid pressure at a point in a liquid at rest is directly proportional to the depth of the point below the surface.
   A solid cylinder of diameter 2 inches and of height 5 inches is suspended by a vertical string so that it is totally immersed in a liquid of specific gravity 0·9. The axis of the cylinder is vertical, and the top of the cylinder is 2 inches below the surface. Calculate the total thrust of the liquid

   (i) on the top of the cylinder,
   (ii) on the base of the cylinder.

                   (Weight of 1 cb. ft. water = $62\frac{1}{2}$ lb.).