

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 27 JUNE - MORNING, 9.30 – 12.00

2458

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2 .

Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

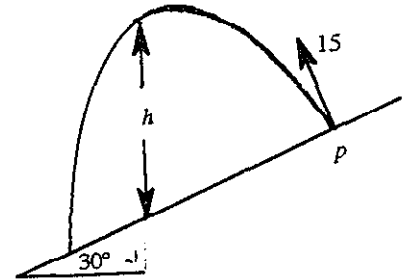
1. (a) A particle, moving in a straight line, accelerates uniformly from rest to a speed $v \text{ m/s}$. It continues at this constant speed for a time and then decelerates uniformly to rest, the magnitude of the deceleration being twice that of the acceleration. The distance travelled while accelerating is 6 m . The total distance travelled is 30 m and the total time taken is 6 s .
- (i) Draw a speed-time graph and hence, or otherwise, find the value of v .
- (ii) Calculate the distance travelled at $v \text{ m/s}$.
- (b) Two points p and q are a distance d apart. A particle starts from p and moves towards q with initial velocity $2u$ and uniform acceleration f . A second particle starts at the same time from q and moves towards p with initial velocity $3u$ and uniform deceleration f . Prove that
- (i) the particles collide after $\frac{d}{5u}$ seconds,
- (ii) if the particles collide before the second particle comes to instantaneous rest, then
- $$fd < 15u^2,$$
- (iii) if $fd = 30u^2$ then the second particle has returned to q before the collision.
2. To a man walking North at 4 m/s a constant wind appears to have a speed of 3 m/s . The man then changes direction and when he is walking West at 3 m/s the wind appears to have a speed of 4 m/s . There are two possible wind velocities.
- If \vec{i} and \vec{j} are unit vectors in the East and North directions, respectively:
- (i) show that $-3\vec{i} + 4\vec{j}$ is one possible wind velocity,
- (ii) find the second possible wind velocity.

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3. (a) A golf ball, at rest on horizontal ground, is struck so that it starts to move with velocity $3u\mathbf{i} + u\mathbf{j}$ where \mathbf{i} and \mathbf{j} are unit vectors along and perpendicular to the ground, respectively. In its flight the ball rises to a maximum height of 15 m. Calculate

- (i) the value of u ,
- (ii) the magnitude and direction of the velocity with which the ball strikes the ground.

- (b) A particle is projected from a point p with initial speed 15 m/s, down a plane inclined at an angle of 30° to the horizontal. The direction of projection is at right angles to the inclined plane. (The plane of projection is vertical and contains the line of greatest slope). Find

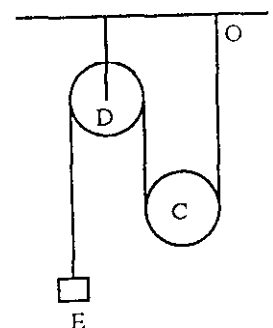


- (i) the perpendicular height of the particle above the plane after t seconds and hence, or otherwise, show that the vertical height h of the particle above the plane after t seconds is
- $$10\sqrt{3}t - 4.9t^2$$
- (ii) the greatest vertical height it attains above the plane (i.e. the maximum value of h) correct to two places of decimals.

4. (a) A particle A, of mass m kg, rests on a smooth horizontal table. It is connected by a light inextensible string which passes over a light, smooth, fixed pulley to a second particle B, of mass 2 kg, which hangs freely under gravity. The system starts from rest with A at a distance of 1 metre from the pulley.

- (i) Calculate the acceleration of A.
- (ii) If A reaches the pulley in $\frac{5}{7}$ seconds, find m .

- (b) The diagram shows a light inextensible string having one end fixed at O, passing under a smooth movable pulley C of mass km kg and then over a fixed smooth light pulley D. The other end of the string is attached to a particle E of mass m kg.



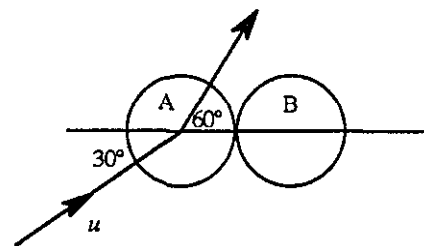
- (i) Show on separate diagrams the forces acting on each mass when the system is released from rest.
- (ii) Show that the upward acceleration of C is $\frac{(2-k)g}{4+k}$.
- (iii) If $k = 0.5$, find the tension in the string.

5. (a) A smooth sphere P, of mass m , moving with speed ku collides directly with a smooth sphere Q, of mass km , moving in the same direction with speed u . P is brought to rest by the impact.

(i) Find the velocity of Q after the collision in terms of u .

(ii) Prove that $k \geq 3$.

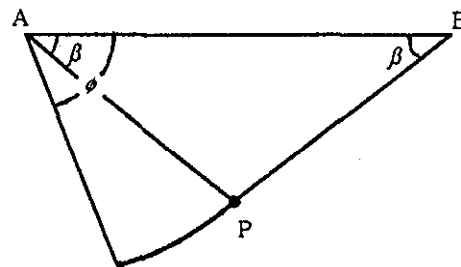
- (b) A smooth sphere A, of mass m , moving with speed u collides with a smooth sphere B, of mass m , which is at rest. The direction of motion of A before impact makes an angle of 30° with the line of centres. If the coefficient of restitution between the spheres is e , find



(i) the velocity of each sphere after impact

(ii) the value of e if after impact the direction of A makes an angle of 60° with the line of centres.

6. (a) A particle P, of mass m , is suspended by two inextensible light strings PA, PB, of equal length where A and B are fixed at the same horizontal level and each string is inclined at an angle β to the horizontal.



(i) Find the tension in string PA.

(ii) If the string PB is cut so that P starts to move in a circular path, prove that the tension in the string PA when it makes an angle ϕ with the horizontal is

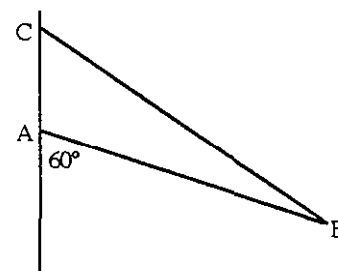
$$mg(3 \sin \phi - 2 \sin \beta).$$

(iii) If the tension in PA is suddenly halved when PB is cut, find the angle β .

7. (a) (i) A particle of weight W is attached to two light inextensible strings each of length 26 cm. The other ends of the strings are attached to two points on the same horizontal level 48 cm apart. Find the tension in each of the strings in terms of W .

(ii) The strings are replaced by two light elastic strings, each of natural length 26 cm and elastic constant k , attached in the same way. The position of equilibrium of the particle is now 8 cm below its previous position of equilibrium. Find the value of k in terms of W .

- (b) A uniform rod AB, of length 57 cm and weight W , rests in a vertical plane at an angle of 60° with the downward vertical. A is in contact with a smooth vertical wall and B is supported by a light inextensible string BC where C is a point on the wall vertically above A. Find



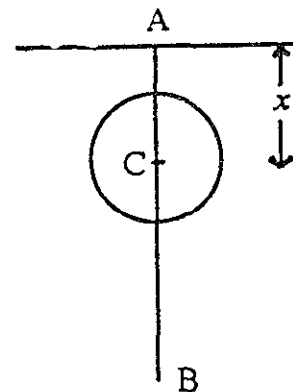
(i) the tension in the string in terms of W

(ii) the length of the string correct to the nearest cm.

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8. (a) Prove that the moment of inertia of a uniform circular disc, of mass m , and radius r , about an axis through its centre perpendicular to its plane is $\frac{1}{2} mr^2$.

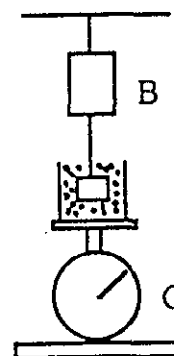
- (b) A thin uniform rod AB of mass m , and length $2a$ can turn freely in a vertical plane, about a fixed horizontal axis through A. A uniform circular disc of mass $24m$ and radius $a/3$ has its centre C clamped to the rod so that the length AC = x and the plane of the disc passes through the axis of rotation.



- (i) Show that the moment of inertia of the system about the axis is $2m(a^2 + 12x^2)$.
- (ii) The system makes small oscillations. Find the period and show that the period is a minimum when $x = a/4$.

9. (a) What is the area of the underside of the smallest rectangular block of ice 30 cm thick that will support a man of mass 80 kg? The relative density of the ice is 0.917 and it is floating in sea-water of relative density 1.03.

- (b) A block hangs from a spring balance B and is submerged in a liquid contained in a beaker. The mass of the beaker is 1 kg, and the mass of the liquid is 1.5 kg. Balance B reads 2.5 kg and balance C reads 7.5 kg. The volume of the block is 0.01 m^3 . Calculate



- (i) the mass of the block
- (ii) the relative density of the liquid.

10. (a) If

$$\frac{dy}{dx} + x + xy = 0$$

and $y = 2$ when $x = 0$, find, correct to two places of decimals, the value of y when $x = 1$.

- (b) A particle moves in a straight line and undergoes a retardation of $0.04v^3 \text{ m/s}^2$, where v is its speed.
- (i) If the initial speed of the particle is 25 m/s, find its speed when it has travelled a distance of 49 m.
- (ii) Find the time for the speed to reduce from 25 m/s to 15 m/s.