M. 32

2259

AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1995

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 23 JUNE – MORNING, 9.30 to 12.00

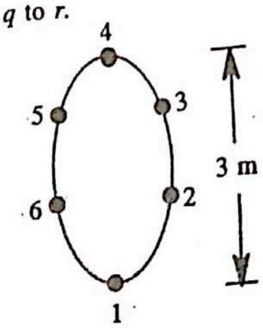
Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2 .

Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

1. *(a)* A particle moving in a straight line with constant acceleration passes three points p, q, r and has speeds u and $7u$ at p and r respectively.

   (i) Find its speed at q the mid-point of $[pr]$ in terms of u.
   (ii) Show that the time from p to q is twice that from q to r.

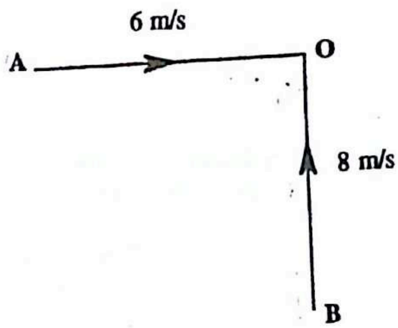
   (b) A juggler throws up six balls, one after the other at equal intervals of time t, each to a height of 3 m. The first ball returns to his hand t seconds after the sixth was thrown up and is immediately thrown to the same height, and so on continually. (Assume that each ball moves vertically).



   Find

   (i) the initial velocity of each ball.
   (ii) the time t.
   (iii) the heights of the other balls when any one reaches the juggler's hand.

2. *(a)* Two particles A and B are moving along two perpendicular lines towards a point O with constant velocities of 6 m/s and 8 m/s respectively. When A is 64 m from O, B is 62 m from O.



   (i) Find the distance of each particle from O after t seconds.

   (ii) Hence, or otherwise, find the times at which their distance apart is 50 m.

   (b) A girl wishes to swim across a river 60 m wide. The river flows with a velocity of q m/s parallel to the straight banks and the girl swims at a velocity of p m/s relative to the water. In crossing the river as quickly as possible she takes 100 s and is carried downstream 45 m.

   Find

   (i) p and q.
   (ii) how long will it take her to swim in a straight line back to the original starting point.

OVER→

3. *(a)* A ball kicked from a point p on level ground hit the ground for the first time 27 m from p after a time 3 s. The ball just passed over a wall standing 5.4 m from p.

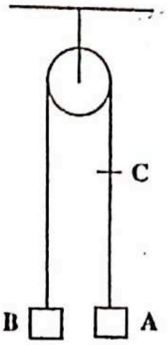
Find

(i) the horizontal and vertical components of its initial velocity.
(ii) the height of the wall.
(iii) the speed of the ball as it passed over the wall.

(b) A plane is inclined at an angle of 30° to the horizontal. A particle is projected up the plane with initial velocity 20 m/s at an angle θ to the plane. The plane of projection is vertical and contains the line of greatest slope. If the particle strikes the plane at right angles,

(i) find the angle of projection θ .
(ii) prove that if the angle of projection is increased to 45° then the particle strikes the plane obtusely and bounces back down the plane.

4. Two particles A and B of mass 0.4 kg and 0.5 kg respectively are connected by a light inextensible string which passes over a smooth pulley. When A has risen for 1 second, it passes a point C and picks up a mass of 0.2 kg.



Find

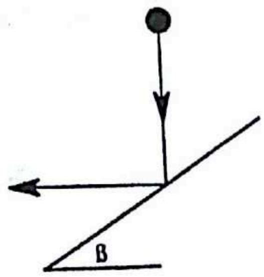
(i) the initial acceleration.
(ii) the velocity of A just before it picks up the mass at C.
(iii) using the principle of conservation of momentum, or otherwise, the velocity of A after picking up the mass at C.
(iv) the distance of A from C at the first position of instantaneous rest.

5. *(a)* Two smooth spheres of masses $2m$ and $3m$ respectively lie on a smooth horizontal table. The spheres are projected towards each other with speeds $4u$ and u respectively.

(i) Find the speed of each sphere after the collision in terms of e, the coefficient of restitution.

(ii) Show that the spheres will move in opposite directions after the collision if $e > \frac{1}{3}$.

(b) A ball falls vertically and strikes a smooth fixed plane. The plane is inclined at an angle β to the horizontal ($\beta < 45°$). The ball rebounds horizontally.



(i) Prove that $\tan \beta = \sqrt{e}$, where e is the coefficient of restitution.

(ii) Show that the fraction of kinetic energy lost during impact is $(1 - e)$.

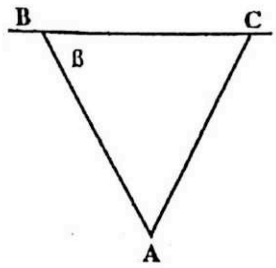
6. *(a)* A light string [*op*], of length *l*, is fixed at the end *o*, and is attached at the other end *p* to a particle which is moving uniformly in a horizontal circle whose centre is vertically below and distant *h* from o.

Prove that the period of the motion is $2\pi\sqrt{\frac{h}{g}}$.

(b) A particle of mass *m*, attached to a fixed point by a light inelastic string, describes a circle in a vertical plane. The tension of the string when the particle is at the highest point of the orbit is T_1 and when at the lowest point it is T_2. Prove that

$$T_2 = T_1 + 6mg.$$

7. Two uniform rods AB and AC of equal length and of weights 2W and W respectively are smoothly hinged together at A and hinged at B and C to a horizontal beam. The rods are in a vertical plane with A below BC.



Prove that

(i) the horizontal and vertical components of the reaction of the hinge at A on the rod AB are

$$\frac{3W}{4\tan\beta} \quad \text{and} \quad \frac{W}{4}$$

respectively, where ß is the angle ABC.

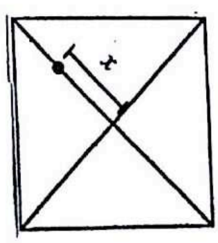
(ii) if the total reactions at B and C are perpendicular to each other, then

$$\tan \beta = \frac{3}{\sqrt{35}}.$$

8. *(a)* Prove that the moment of inertia of a uniform square lamina, of mass *m* and side 2*a*, about an axis through its centre parallel to one of its sides is $\frac{1}{3}ma^2$.

(b) A uniform square lamina of side 2*a* is freely pivoted at a point in one diagonal and oscillates in its own plane. Prove that when the period of small oscillations is a minimum the distance of the pivot from the centre is *x* where

$$3x^2 = 2a^2.$$



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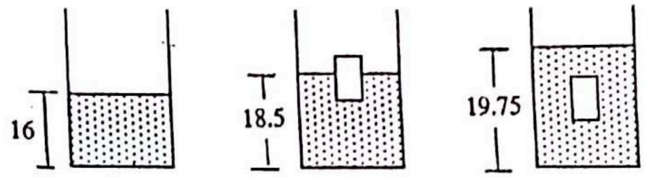
9. *(a)* What quantity of water must be mixed with 1 litre of milk to reduce its relative density from 1.03 to 1.02?

(b) A cylinder contains water to a height of 16 cm. A body of mass 0.02 kg is placed in the cylinder. It floats and the water level rises to 18.5 cm. The body is then completely submerged and the water level rises to 19.75 cm.



Find

(i) the relative density of the body.

(ii) the force required to submerge the body.

(iii) the volume of water in the cylinder.

10. *(a)* Solve the differential equation

$$(1 + x^2)\frac{dy}{dx} = \frac{4}{y}$$

if $x = 0$ when $y = 1$.

(b) A particle of mass m falls from rest against an air resistance of mkv, where k is constant and v is the speed. Prove that

(i) the time taken to acquire a speed of $\frac{g}{2k}$ is $\frac{\ln 2}{k}$.

(ii) the speed of the particle tends to a limit $\frac{g}{k}$.