LEAVING CERTIFICATE EXAMINATION, 1993

1225

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 25 JUNE – MORNING, 9·30 to 12·00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 m/s^2 .
Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

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1. *(a)* A particle moving in a straight line travels 30 m, 54 m and 51 m in successive intervals of 4, 3 and 2 seconds.

(i) Verify that the particle is moving with uniform acceleration.

(ii) Draw an accurate speed-time graph of the motion.

(b) A particle P is projected vertically upwards from the ground with an initial velocity of 47 m/s. Two seconds later another particle Q is projected vertically upwards from the same point with initial velocity 64·6 m/s. Calculate

(i) how long Q is in motion before it collides with P.

(ii) the height at which the collision occurs.

2. *(a)* A girl travelling south at 11 m/s finds that the wind appears to blow from the East but on doubling her speed it appears to come from the South-east.

Calculate the magnitude and direction of the wind's velocity.

(b) A boy who can swim at 5/9 m/s wishes to cross a river 50 m wide, flowing at 5/6 m/s, as quickly as possible. Calculate

(i) the direction he should take.

(ii) the time he takes to cross.

(iii) how far downstream from his starting point he goes.

OVER →

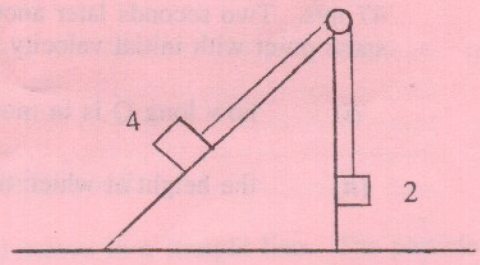
3. *(a)* A particle is projected on a horizontal plane with initial velocity u at an angle β to the horizontal. If the range of the projectile is three times the greatest height, prove that $\tan \beta = 4/3$.

(b) A particle is projected up an inclined plane with initial speed u. The line of projection makes an angle of 30° with the plane and the plane is inclined 30° to the horizontal. (The plane of projection is vertical and contains the line of greatest slope.) The particle strikes the plane at an angle θ, $\theta < 90°$ (i.e. θ is the landing angle).

(i) Express the velocity and displacement of the particle after t seconds in terms of unit vectors $\vec{i}$ and $\vec{j}$ along and perpendicular to the plane, respectively.

(ii) calculate θ.

4. A smooth wedge, of mass 10 kg and slope $\tan^{-1} 3/4$, is placed on a smooth horizontal surface. A particle of mass 4 kg is placed on one face and it is connected by a light inextensible string which passes over a light frictionless pulley to a second particle of mass 2 kg which hangs vertically and touches the side of the wedge.



(i) Show, on separate diagrams, the forces acting on the wedge, the 4 kg mass and the 2 kg mass.

(ii) Prove that the acceleration of the wedge is g/67.

5. State the laws governing the oblique collision between two smooth elastic spheres.

A smooth sphere A, of mass m, moving with speed u, collides with a smooth sphere B, of mass m, which is at rest. The direction of motion of A before and after impact makes angles $\cos^{-1}\frac{5}{\sqrt{35}}$ and $\cos^{-1}\frac{3}{7}$ respectively with the line of centres. The coefficient of restitution between A and B is 2/5. Show that after impact, A and B have the same speed.

Calculate the loss of kinetic energy due to the impact.

6. *(a)* A particle of mass m moves with simple harmonic motion under the action of a variable force. If the maximum value of the force is $\frac{7m}{16}$ and the amplitude of the motion is 4 m calculate

(i) the period of the oscillation.

(ii) the speed of the particle at a time $\frac{2\pi}{\sqrt{7}}$ seconds after passing through the centre of oscillation.

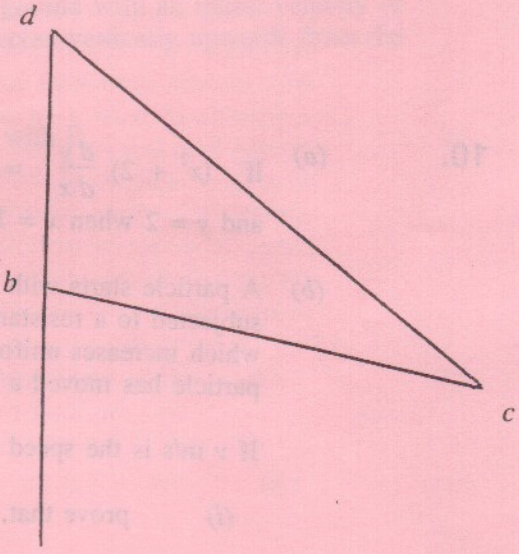
(b) A light elastic string, of elastic constant $\frac{48mg}{l}$ and natural length l has one end attached to a fixed point. Two particles of masses $3m$ and $2m$ are attached to the other end and the system hangs in equilibrium. If the $2m$ mass falls off

(i) prove that the $3m$ mass will move with simple harmonic motion of period

$$\frac{\pi}{2}\sqrt{\frac{l}{g}}$$

(ii) find the amplitude of the motion.

7. *(a)* A uniform rod bc, of weight W and length $2l$, rests in equilibrium with b in contact with a rough vertical wall. One end of a light inextensible string is fixed to a point d on the wall vertically above b, the other end is attached to c.



(i) If $|\angle bdc| = 30°$ and $|\angle dbc| = 120°$, prove that $\mu \geq 1/\sqrt{3}$
where μ is the coefficient of friction between the rod and the wall.

(ii) If $\mu = \frac{1}{\sqrt{3}}$ and the string is shortened so that the $|\angle bdc| = \theta$, where $\theta > 30°$, prove that the rod slips down the wall.

8. *(a)* Prove that the moment of inertia of a uniform rod of mass m and length $2l$, about an axis through its centre of mass perpendicular to the rod is $\frac{1}{3}m^2$.

(b) A uniform rod of mass m and of length 1·2 m swings in a vertical plane about a horizontal axis through the rod at a distance of 0·4 m from its upper end.

(i) If v m/s is the velocity of the lower end when the rod is vertical, prove that the rod will make a complete revolution if $v \geq 5{\cdot}6$ m/s.

(ii) If a mass m is attached to each end of the rod and the compound body is set in motion calculate the period of small oscillations correct to two decimal places.

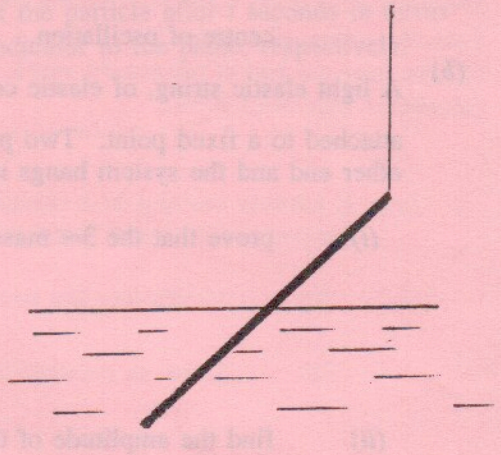
OVER →

9. *(a)* A solid, uniform cylinder 20 cm long floats in water with its axis vertical and 17 cm of its length immersed. Oil of relative density 0·8 is poured on to the water until the top of the cylinder is in the oil surface.
What is the depth of the layer of oil?

(b) A thin uniform rod, floats motionless in water in an inclined position. The upper end of the rod is supported by a string.

(i) Prove that the string is vertical.

(ii) If the relative density of the rod is 0·64, calculate the length of the immersed part of the rod.



10. *(a)* If $(x^2 + 2)\dfrac{dy}{dx} = x(y + 1)$
and $y = 2$ when $x = 1$, find the value of y when $x = 2$.

(b) A particle starts with a speed of 20 m/s and moves in a straight line. The particle is subjected to a resistance which produces a retardation which is initially 8 m/s^2 and which increases uniformly with the distance moved, having a value of 9 m/s^2 when the particle has moved a distance 5 m.

If v m/s is the speed of the particle when it has moved a distance x m

(i) prove that, while the particle is in motion,

$$v\frac{dv}{dx} = -\left(8 + \frac{x}{5}\right)$$

(ii) calculate the distance moved by the particle in coming to rest.