David Malone

AN ROINN OIDEACHAIS M. 32

LEAVING CERTIFICATE EXAMINATION, 1992

001061

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 26 JUNE – MORNING, 9·30 to 12·00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 m/s^2 .
Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

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1. *(a)* A balloon ascends vertically at a uniform speed.
7·2 seconds after it leaves the ground a particle is let fall from the balloon.
The particle takes 9 seconds to reach the ground.
Calculate the height from which the particle was dropped.

(b) Two particles P and Q are moving in the same direction along parallel straight lines with accelerations 5 m/s^2 and 4 m/s^2, respectively. At a certain instant P has a velocity 1 m/s and Q is 25·5 m behind P moving with velocity 11 m/s.

(i) Prove that Q will overtake P and that P will in turn overtake Q.

(ii) When Q is in front of P find the greatest distance between the particles.

2. An aeroplane having a speed of 500 km/h in still air travels 1500 km due North when the wind is blowing from 60° East of North at 90 km/h, and then returns to the starting point along the same path. Calculate

(i) the directions in which the aeroplane must travel on the outward and return journeys.

(ii) the total time taken

(iii) the total time taken if there was no wind blowing.

3. A particle is projected with velocity $6\sqrt{g}$ m/s , at an angle α to the horizontal, from a point 18 m in front of a vertical wall 5·5 m high.

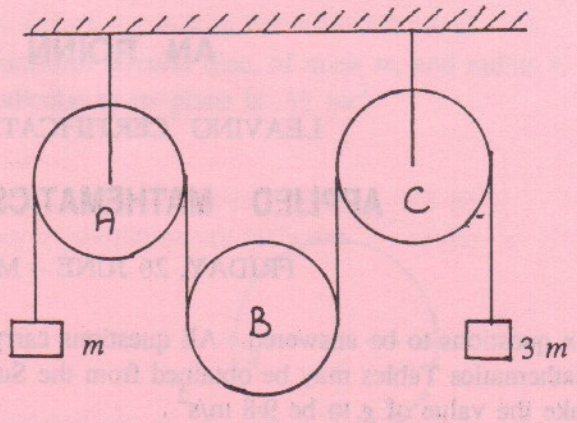
(i) Calculate the two possible values of α which will enable the particle to just clear the wall.

(ii) Show that the value of α is $\tan^{-1} 2$ for maximum clearance height.

OVER→

4.

A light inextensible string passes over a fixed pulley A, under a movable pulley B, of mass M, and then over a second fixed pulley C. A mass m is attached to one end of the string and a mass $3m$ is attached to the other end. If the system is released from rest



(i) Show in a diagram the forces acting on each of the three masses.

(ii) Prove that the tension, T, of the string is given by the equation

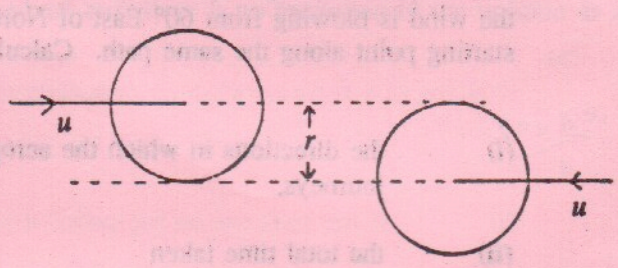
$$T\left(\frac{1}{M} + \frac{1}{3m}\right) = g .$$

(iii) Show that if $M = 3m$ then pulley B will remain at rest while the two masses are in motion.

System is frictionless

5.

Two equal smooth spheres, of radius r, move horizontally in opposite directions with speed u. Their centres lie on two parallel lines a distance r apart. The coefficient of restitution is $\frac{1}{3}$.



(i) Prove that at the moment of impact the line of centres makes an angle of 30° with the previous direction of motion.

(ii) Find the velocity of each sphere after impact.

(iii) What fraction of the kinetic energy is lost as a result of the collision.

6. *(a)* If the displacement of a moving particle at any time t is given by the equation

$$x = 5\cos\omega t + 12\sin\omega t$$

(i) show that the motion is a simple harmonic motion.

(ii) calculate the amplitude of the motion.

(b) A particle of mass 2 kg is attached to one end of a light elastic string of natural length 1 m and elastic constant 14 N/m. The other end of the string is fixed to a point A on a smooth horizontal table. The particle is pulled across the table and released from rest at a point C which is a distance 1·5 m from A.
If B is a point on AC such that |AB| = 1m,

(i) prove that the particle performs simple harmonic motion when travelling from C to B.

(ii) calculate the time taken to travel from C to B.

(iii) prove that the particle then travels for $\frac{4}{\sqrt{7}}$ s with constant speed.

7. *(a)* A body of weight 14 N is kept at rest on a smooth plane of inclination α by a horizontal force of 7N together with a force of 7N acting up along the line of greatest slope of the plane.

Show that $\cos\alpha = \frac{3}{5}$.

(b) Two equal uniform rods pq and qr, each of weight W, are freely jointed at q. The rods are in a vertical plane and the ends p and r rest on uniformly rough horizontal ground.

(i) If the rods are on the point of slipping, prove

$$\mu = \frac{1}{2\tan\alpha}$$

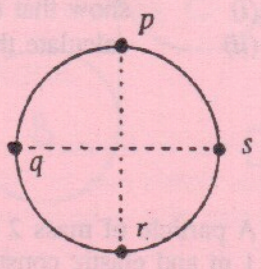
where μ is the coefficient of friction and α is the inclination of each rod to the horizontal.

(ii) If $\mu > \frac{1}{2\tan\alpha}$ and $\mu\tan\alpha < 1$, find the maximum weight which can be placed at q without slippage taking place.

OVER→

8. *(a)* Prove that the moment of inertia of a uniform circular disc, of mass m, and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2} mr^2$

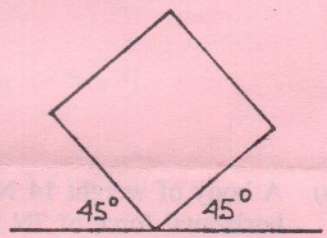
(b) Particles, each of mass m, are fixed at q , r and s which are on the circumference of a uniform circular disc of mass 8m and radius r . p , q , r , and s are the extremities of two perpendicular diameters. The system can turn freely in a vertical plane about p. Calculate the period of small oscillations.



9. *(a)* State the Principle of Archimedes.

A buoy in the form of a hollow spherical shell of external radius 0·8 m and internal radius r floats in water with half of its volume immersed. If the relative density of the material of the shell is 5·12, calculate the thickness of the shell.

(b) A closed tank in the shape of a cube of side $\sqrt{2}$ metres is half full with water and half with oil of relative density 1·2. The tank is placed on a horizontal table and is then tilted about one edge until the faces about this edge are inclined at 45 ° to the horizontal.



Find the thrust on one of the vertical faces of the tank.

(i) if the oil and water are not mixed.

(ii) if the oil and water are mixed and there is no reduction in the volume as a result of mixing.

10. *(a)* If

$$\frac{dy}{dx} = 2\sin 2x + \cos 4x$$

and if $y = 1$ when $x = \frac{\pi}{4}$, find the value of y when $x = \frac{\pi}{2}$.

(b) A particle experiences a retardation of kv m/s^2 when its velocity is v m/s. Its velocity is reduced from its initial value of 210 m/s to 70 m/s in 0·5 s and it travels a distance x m in this time.

(i) Find the value of k and deduce an expression for the velocity at any time t .

(ii) Caluclate the value of x .