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SCRÚDÚ ARDTEISTIMÉIREACHTA, 1973

APPLIED MATHEMATICS - HIGHER LEVEL

(400 marks)

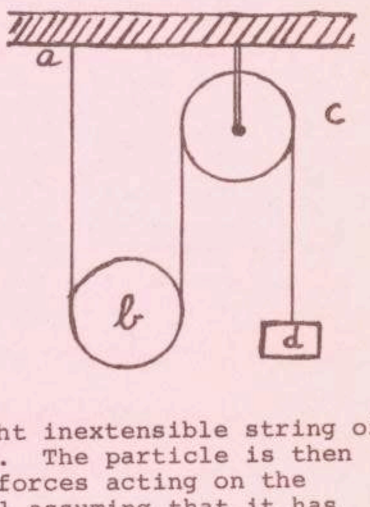
TUESDAY, 26 JUNE - MORNING, 9.30 to 12

Not more than six questions may be answered.
All questions are of equal value.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres second^2. $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. A cyclist has a maximum acceleration of 2 m/s^2, a maximum speed of 15 m/s and a maximum deceleration of 4 m/s^2. The cyclist wishes to travel a distance s from rest to rest in the shortest time. Find the time taken in the two cases (i) s = 105 m and (ii) s = 54 m. Draw a rough velocity-time graph for each case and explain why $84\frac{3}{8}$ m is a critical distance.

2. A perfectly elastic particle falls vertically with speed u on to a smooth plane inclined at an angle α to the horizontal, and rebounds, hopping down the plane. Write down its displacement $\vec{r}$ from the point of contact after time t in terms of $\vec{i}$ and $\vec{j}$, where $\vec{i}$ is drawn directly down the plane. Show that the length of the first hop is $\frac{4u^2 \sin\alpha}{g}$ and that the length of the second hop is double this.

3. A ship A is steaming with velocity v m/s where $\vec{v} = 5\sqrt{3}\ \vec{i} + 5\vec{j}$ where $\vec{i}$ and $\vec{j}$ are pointing East and North, respectively. At midday a second ship B has a position $10\vec{i}$ km with respect to A. Find the minimum speed B must have if it is to intercept A. If the maximum speed of B is in fact 6 m/s, show that it can steer in either of two directions to intercept A and calculate the two times of interception.



4. The diagram shows a light inelastic string with one end connected to a fixed point a of a ceiling, passing under a heavy movable pulley b of mass 10 kg and then over a fixed pulley c attached to the ceiling. To the other end d of the string is attached a particle of mass 10 kg hanging freely. Show in separate diagrams the forces acting on the particle and on the pulley b when they are released from rest. Show that the acceleration of d is double that of b, and calculate the acceleration of d and the tension in the string. (Neglect the inertia of both pulleys).

5. A particle of mass 10 kg hangs freely from the end b of a light inextensible string of length 0·2 m which is attached at the other end to a fixed point o. The particle is then projected horizontally with speed 1·4 m/s. Show in a diagram the forces acting on the particle when ob is inclined at an angle θ to the downward vertical assuming that it has a speed v m/s at that point. Use conservation of energy to determine v and express the tension in the string in terms of θ. Find where the particle comes to instantaneous rest and show that the tension in the string is then 49N.

6. A light string $rstu$ is attached to fixed points at its ends r and u, so that u is vertically below r. Particles of weights 50 N and 100 N are attached to the string at s and t, respectively, and a horizontal force x newtons is applied to the particle at s so that the string is in equilibrium in a vertical plane through ru with $\angle sru = \angle rut = 45^\circ$ and $\angle rst = 105^\circ$. Show in separate diagrams the forces acting on the two particles and prove that $x = (250 + 100\sqrt{3})$ N.

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7. An equilateral triangle pqr is formed from three uniform rods, each of length $2a$ and weight W, freely jointed at their ends. The triangle is freely suspended by a string attached to the midpoint o of pq so that it hangs symmetrically under gravity with r vertically below o. Show in separate diagrams the forces acting on pq and pr, and calculate the horizontal and vertical components of the reactions at r.

8. Prove that the moments of inertia of a uniform circular disc of radius 0·4 m and mass 5 kg about an axis oq through its centre o and perpendicular to the disc is 0·4 kg m^2.

Such a disc can rotate freely about the axis oq which is fixed horizontally. A light inextensible string is wound around the rim of the disc with one end attached to it, and to the other end is tied a particle p of mass 2 kg which hangs vertically. If the system is released from rest, show that the speed of p is 2·8 m/s after it has descended a distance 0·9 m.

9. An engine pulls a train along a level track against a resistance which at any time is K times the momentum. The engine works at constant power $25KMU_0{}^2$, where M is the total mass of the train and engine and K, U_0 are constants. Show that the equation of motion of the train is $v\,\dfrac{dv}{dt} = K\,(25U_0{}^2 - v^2)$, and find the time taken to increase speed from U_0 to $4U_0$.

10. State the Principle of Archimedes.

A solid hemisphere of radius a is held submerged in a liquid of density ρ with its plane face horizontal and uppermost at a distance $2a$ below the free surface of the liquid. Calculate the magnitude, direction and line of action of

(i) the force exerted by the liquid on the plane face,

(ii) the total force exerted by the liquid on the surface of the solid.