AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1970

APPLIED MATHEMATICS - HONOURS

TUESDAY, 23rd JUNE - Afternoon, 2 to 4.30

Not more than six questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Supereintendent.

1. A bullet of mass m is fired with speed v into a fixed block of wood and is brought to rest in a distance d. Find the resistance to motion assuming it to be constant.

Another bullet also of mass m is then fired with speed $2v$ into another fixed block of thickness $2d$, which offers the same resistance as the first block. Find the speed with which the bullet emerges, and the time it takes to pass through the block.

2. Prove that the formula $x = ut + \frac{1}{2}at^2$ represents the distance x travelled in time t by a body moving in a straight line with constant acceleration a.

A train takes $4\frac{1}{2}$ minutes to travel between two stations S_1 and S_2 which are 4500 meters apart. It starts from rest at S_1 and finishes at rest at S_2, by travelling with uniform acceleration for the first minute and with uniform deceleration for the last $\frac{1}{2}$ minute. Find the train's constant speed during the remainder of the journey.

If a second train, travelling with a constant speed of 1000 m/min, in the same direction passes S_1 as the first train leaves this station, find when overtaking occurs. (The lengths of the trains may be neglected).

3. A particle is projected under gravity with an initial velocity v_0 at an angle θ to the horizontal. Find its position and the direction of motion after time T in terms of v_0, θ, g and T.

A particle is projected from the top of a cliff which is 425 ft. above sea level and the angle of projection is 45° to the horizontal. If the greatest height reached above the point of projection is 200 ft, find the speed of projection and the time taken to reach this greatest height.

Find when and where the particle strikes the sea.

(Take g to be 32 ft/sec^2).

4. Prove that the bob of a simple pendulum moves in simple harmonic motion - stating any assumptions made.

The string of such a pendulum is 2 ft. long and the bob is released from rest when at a distance $\frac{1}{4}$ ft from the equilibrium position. Calculate the time taken to travel halfway to the equilibrium position and the speed of the bob then.

(Take g to be 32 ft/sec^2).

5. By deriving an expression for the necessary acceleration, prove that a particle of mass m moving in a circle of radius r with speed v must have a force of magnitude $\frac{mv^2}{r}$ pointing towards the centre acting on it.

A particle of mass 4 lbs. moving on the inside smooth surface of a fixed spherical bowl of radius 2 ft is describing a horizontal circle of radius $\sqrt{3}$ ft. Find the constant speed of rotation and the reaction of the sphere on the particle.

(Take g to be 32 ft/sec^2).

6. Show that the centre of gravity of a uniform triangular lamina coincides with the centre of gravity of three equal particles placed at the vertices of the triangle.

Hence find the centre of gravity of a uniform trapezium ABCD, of weight W, in which 2 AB = CD and AD = BC.

A particle of weight w is attached at D and the system is suspended by a string attached to the midpoint F of CD. If in the position of equilibrium F is vertically above A show that $w = \frac{2W}{9}$.

7. A particle of mass 10 lbs. is placed on a rough inclined plane. The least force acting up the plane which will prevent the particle slipping down the plane is 2 lbs. weight. The least force acting up the plane which will make the particle slip upwards is 10 lbs wt. Show that the coefficient of friction is $\frac{1}{2}$ and that the inclination of the plane is α where $\sin\alpha = \frac{3}{5}$.

Find the least force required to move the particle up the plane.

8. Two equal uniform rods AB and BC each of weight W are freely jointed at B. The system is suspended freely from A and a horizontal force $\frac{W\sqrt{3}}{2}$ is applied at the lowest point C. If in the equilibrium position the inclination of AB to the downward vertical is 30°, find the corresponding inclination of BC and the supporting force at A.

9. A small uniform cylinder of density ρ, mass m, total length l and uniform cross section floats in a liquid of density 2ρ with its axis vertical. Find the thrust on the cylinder when it is displaced vertically in the liquid, without being completely immersed, through a distance x from the equilibrium position. Show that if it is released in this position, it will oscillate with simple harmonic motion of period $2\pi\sqrt{\frac{l}{2g}}$.