LEAVING CERTIFICATE EXAMINATION, 1969

APPLIED MATHEMATICS - HONOURS

TUESDAY, 24th JUNE - Afternoon, 2 to 4.30

Not more than six questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

1. State the principle of the conservation of momentum.
A car weighing 2 tons and moving at 60 m.p.h. along a main road collides with a lorry of weight 10 tons which emerges at 30 m.p.h. from a crossroad at right angles to the main road. If the two vehicles lock, what will be their velocity after the collision ?

2. A helicopter flies from an aerodrome to a place due North, a distance 119 kilometers from the aerodrome, and then returns. The speed of the helicopter in still air is 52 meters/sec. and there is a wind of speed $20\sqrt{2}$ meters/sec. blowing from the North-East throughout the journey. Show that the actual speed of the helicopter on the outward journey is 28 meters/sec. and calculate the total time for both journeys.

3. (i) Two identical uniform rods AB and BC are rigidly connected at right angles at B. Find the centre of gravity of the compound body. The body is freely suspended from A. Find the inclination of AB to the vertical.

(ii) A thin uniform wire is bent into the shape of a quadrilateral PQRS. The coordinates of the vertices are P(3, 0), Q(8, 12), R(5, 16) and S(0, 4). Find the coordinates of the centre of gravity of the wire.

4. A projectile is fired with an initial speed of 24 ft./sec. to hit a target at a horizontal distance of 9 ft. from the point of projection and at a vertical height of $4\frac{1}{2}$ ft. Find the two possible angles of projection and the ratio of the times of flight along the two paths. Find the speed of the projectile at impact in each case.

(Take g to be 32 ft./sec.^{2}).

5. A train of mass 100 tons and of constant 1120 horse-power, is allowed to run with the engine turned off down a slope of inclination $\sin^{-1}(\frac{1}{30})$ to the horizontal and reaches a maximum speed of 80 m.p.h. Find the resistance to motion in tons weight. If the resistance to motion in all cases is directly proportional to the square of the speed, calculate the acceleration of the train on a level track when its speed is 60 m.p.h.

(Take g to be 32 ft./sec.^{2}).

6. A car A moves along a straight road PQ with constant acceleration of 5 ft/sec.^{2} in the direction PQ and its velocity at P is 10 ft/sec. in the same direction. Three seconds after A has left P another car B starts from P with a velocity of 38 ft/sec. and a uniform acceleration of 4ft/sec.^{2}, both in the direction PQ. When and where will B overtake A ? Show that after passing A, B will never be ahead by more than 32 ft.

7. A particle of mass 3 lbs. is suspended from a fixed point O by a light inelastic string of length 2 feet. It is projected horizontally with speed 8 ft./sec. from a point 2 feet vertically below O. How high does it rise in the subsequent motion ?
When it returns to O it collides and coalesces with a stationary particle of mass 6 lbs. How high does the combined mass rise in the subsequent motion ?
Calculate the greatest tension in the string during the motion of the combined mass.

(Take g to be 32 ft./sec.^{2}).

8. Define simple harmonic motion.
A particle is moving in a straight line with simple harmonic motion. When it is 5 feet from the centre of its path its speed is 24 ft./sec. and when it is 12 feet away its speed is 10 ft./sec. Find the period and amplitude of the motion. Find also the time taken for the particle to travel from the centre of its path to a point where its speed is half the maximum speed.

9. Two solid uniform spheres each of radius 4 cms. are connected by a light string and are completely immersed in a tank of water. The specific gravities of the spheres are $\frac{1}{2}$ and $2\frac{1}{2}$ respectively. Find the tension in the string and the reaction between the bottom of the tank and the heavier sphere.