LEAVING CERTIFICATE EXAMINATION, 1968

APPLIED MATHEMATICS - HONOURS

WEDNESDAY, 26th JUNE - Afternoon, 2.30 to 5

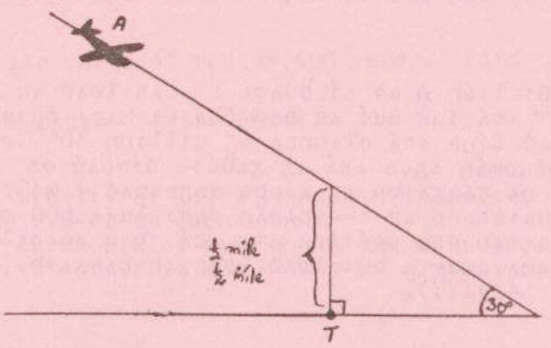
Not more than six questions may be answered. All questions are of equal value.
Mathematical Tables may be obtained from the Superintendent.

1. Show how to find the centre of gravity of a non-rectangular parallelogram.
A thin uniform square sheet of metal ABCD of area 4 square inches weighs 4 ounces. At the corners A, B, C, D weights of 1 oz., 2 oz., 3 oz., and 4 oz., respectively are placed. Locate the centre of gravity of the system.

2. Establish the formulae $v = rw$ and $a = rw^2$ for the velocity and acceleration of a particle moving with uniform speed in a circle of radius r.
Assuming that the moon travels in a circular orbit whose radius is $3{\cdot}8 \times 10^{10}$ cm. and whose centre is the centre of the earth, and completes a revolution every $2{\cdot}4 \times 10^6$ seconds, calculate the ratio of its acceleration g^1 towards the centre of the earth to gravity g at the earth's surface.

3. Two identical cars A and B each 16 feet in length travel with constant velocity 60 feet per second along a straight level road, the front of A being 44 feet directly behind the rear of B. Immediately on reaching a line drawn across the road perpendicular to the direction of motion of the cars each car decelerates at 12 feet per sec^2.
(i) Show that A crashes into B.
(ii) At what distance from the line does the crash occur ?

4. An aeroplane A about to drop a bomb on a small stationary target T on the sea surface descends along a line which is inclined at an angle of 30° to the horizontal and which passes directly over the target at a vertical height of $\frac{1}{2}$ mile. If the aeroplane travels at 600 m.p.h. what duration of time must elapse between the release of the bomb and its impact on the target. (See diagram).



5. Show that the work done by a body in coming to rest is the kinetic energy of the moving body.
A body of mass 20 lbs. sliding on a rough horizontal plane with initial velocity 22 feet sec.$^{-1}$ is brought to rest by friction in 15 feet. Calculate the coefficient of friction.
At what average rate in horse-power was work done on the body in bringing it to rest ?

6. A particle executes simple harmonic motion with a frequency of 60 vibrations per second. The amplitude is 1·5 millimetres.
Find (i) the maximum speed of the particle,
(ii) the maximum acceleration of the particle.
If the cutting-head of an electric razor has mass 10 grams and oscillates with simple harmonic motion at a frequency of 20 vibrations per second, and with an amplitude of 1·5 millimetres, calculate the maximum value of the driving force.

7. A train is rounding a circular curve of radius 440 yards. A bob hangs at one end of a light string which is fixed in the ceiling of a carriage and which is inclined at an angle of 4°18′ to the vertical. Estimate the speed of the train.

8. A fire-engine raises 200 gallons of water per minute from a well 16 feet deep and then pumps it as a jet issuing with a nozzle velocity of 44 feet per sec. At what horsepower is the engine working ?

9. The density of lead is 11·3 grams cm.$^{-3}$ and that of mercury is 13·6 grams cm.$^{-3}$ Explain why a block of lead will float on mercury.
A rectangular uniform block of wood floats on sea water with $\frac{1}{3}$ of its volume above the water-line. What is the volume of the smallest rectangular block of the wood which will float without being totally submerged while supporting a weight of 100 lb. placed vertically above its centre of gravity ?
(Take 1·03 as the specific gravity of sea-water.)