M.48

AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1967

APPLIED MATHEMATICS - HONOURS

THURSDAY, 22nd JUNE - Afternoon, 2.30 to 5

Not more than six questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

1. A body is projected under gravity with initial velocity $\vec{u}$ at an angle θ to the horizontal. Find in terms of u, θ, (a) the maximum height attained, (b) the time required to reach that height, and (c) the distance it has travelled in a horizontal direction on reaching the maximum height.

A man strikes a golf ball which, at the instant it reaches maximum height of 36 feet above the ground, strikes a tree 300 feet away. Assuming the ground to be level, find the magnitude and direction of the initial velocity of the golf ball, to the nearest ft. per sec. (neglecting effects of air resistance).

2. Show that $\frac{1}{2}mv^2$ is the work done on a stationary mass m in order to give it a velocity $\vec{v}$.

A mass of 30 kg. moving East with a velocity of 400 cm. per sec. makes direct impact on a mass of 50 kg. which is at rest. After the impact the 50 kg. mass moves East with a velocity of 200 cm. per sec. Assuming conservation of momentum find

(a) the velocity of the 30 kg. mass after impact,
(b) the kinetic energy lost on impact.

3. Define Simple Harmonic Motion.

(i) If the bob of a pendulum is given a very small displacement x to one side, the string being kept taut, and then released, show that the resulting motion is approximately simple harmonic.

(ii) A bob of mass 36 grams hangs motionless at one end of an elastic string the other end of which is fixed in the ceiling. If the bob is pulled vertically down a distance l until the tension in the string is 40 grams weight and then released, with what velocity will it pass the position of equilibrium assuming its motion about that position is simple harmonic ?

4. $\vec{V}_1$ is a velocity fixed in magnitude and direction. $\vec{V}_2$ is a velocity fixed in magnitude only. If $\vec{V} = \vec{V}_1 + \vec{V}_2$, use vector diagrams to illustrate the situations where

(a) the magnitude of $\vec{V}$ is greatest.
(b) the magnitude of $\vec{V}$ is least.
(c) the magnitude of $\vec{V}$ is neither greatest nor least.

A boat crosses a straight river 200 ft. wide which flows with a uniform velocity of 4 miles per hour. The boat's velocity relative to the water is 5 miles per hour. Calculate to the nearest foot how far down-stream the boat lands if it crosses the stream in such a direction that it reaches the other side in exactly one minute.

5. (i) A rectangular lamina 3 inches long and 2 inches wide is marked off in six equal squares. One corner square is removed. Calculate the position of the centre of gravity of the remainder.

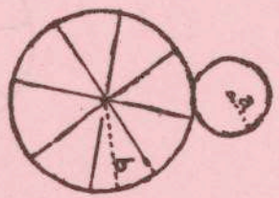
(ii) P is a point outside a triangle XYZ whose centre of gravity is C. Three forces $\vec{PX}$, $\vec{PY}$, $\vec{PZ}$ act at P. Show that their resultant is 3 $\vec{PC}$.

6. A uniform circular hoop of radius a rolls on the outer rim of a fixed wheel of radius b, hoop and wheel being coplanar. (See diagram.) If the angular velocity $\vec{w}$ of the hoop about its centre is constant find

(i) what distance the hoop travels along the circumference of the wheel in time t;

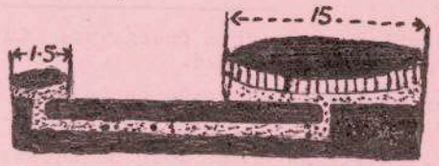
(ii) the velocity and acceleration of the centre of the hoop;

(iii) the velocity of that point of the hoop which is farthest from the wheel.



7. Say what you know about the pressure in a liquid, and the transmission of pressure in a liquid.

The diameter of the small piston of a hydraulic press (see diagram) is 1·5 inches and the diameter of the large piston is 15 inches. What weight on the small piston will support two tons weight on the large piston at the same horizontal level ?



8. Give three properties of friction.

Two light rings can slide on a rough horizontal rod. The rings are connected by a light inextensible string of length l to the mid point of which is attached a weight W. If μ is the coefficient of friction between either ring and the rod, and d is the distance between the rings, show that for equilibrium

$$d \leqslant \frac{\mu l}{\sqrt{1+\mu^2}}.$$

9. A straight light rigid rod of length $2k$ terminated by heavy particles of masses m and M respectively is placed inside a smooth hemispherical bowl of radius r, which is fixed with its rim horizontal. If the particle of mass m rests just below the rim show that

$$mr^2 = M(2k^2 - r^2).$$

OR

9. A train weighing 160 tons is being pulled down a slope of 1 in 80 by an engine weighing 60 tons. The train has velocity 30 m.p.h. and is accelerating at the rate of ½ ft. per sec^2. If the frictional resistance to its motion is 10 lbs. weight per ton find the horse-power at which the engine is working.