AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1966

APPLIED MATHEMATICS – HONOURS

WEDNESDAY, 22nd JUNE – Afternoon, 2 to 4.30

Not more than six questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

1. (a) Represent any two velocities $\vec{v}_1$ and $\vec{v}_2$ by a vector diagram, and illustrate the vectors $(\vec{v}_1 + \vec{v}_2)$ and $(\vec{v}_1 - \vec{v}_2)$.

   (b) When the sun, S, is 30° above the horizon, an aeroplane A glides in for landing with the sun behind, moving with a speed of 80 m.p.h. along a path sloping 15° down from the horizontal, as in the diagram. Find the speed of the plane's shadow on level ground (PQ).
   (Neglect the motion of sun and earth.)
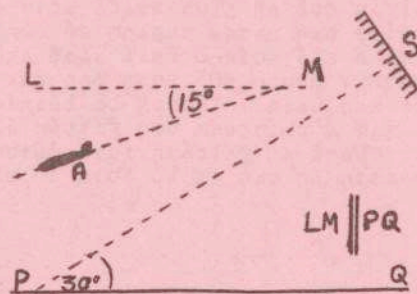
2. Write a short note on linear momentum.
A uniform cube of wood of mass 10 lbs. hangs vertically by a long light string fixed in the centre of one face of the cube. A bullet of mass half an ounce moving horizontally with velocity $\vec{v}$ strikes the centre of another face in a direction perpendicular to the face, embeds itself in the wood and causes the combined mass to swing. If in the first swing the centre of gravity of the combined mass rises 6 inches, calculate the magnitude of $\vec{v}$.

3. For a body moving in a straight line with constant acceleration $\vec{f}$, initial velocity $\vec{u}$ and velocity $\vec{v}$ at time t, show that $v = u + ft$ and $s = ut + \frac{1}{2}ft^2$.
A cage travels down a vertical mine-shaft 750 yards deep in 45 seconds, starting from rest. For the first quarter of the distance only the velocity undergoes a constant acceleration, and for the last quarter of the distance the velocity undergoes a constant retardation, the acceleration and retardation having equal magnitudes. If the velocity, $\vec{v}_1$, is uniform while traversing the central portion of the shaft, find the magnitude of $\vec{v}_1$.

4. The co-ordinates (x,y) of a moving particle at any time t are given by:
$$x = 2\cos^{t}\!/_2, \quad y = 2\sin^{t}\!/_2.$$
Deduce that its motion is circular.
A particle is moving along the circumference of a circle with constant speed. Show that the components of the motion of the particle along any two perpendicular diameters are Simple Harmonic Motions.

5. A particle is projected horizontally from a height with velocity $\vec{v}$ and falls under gravity. If (x,y) are the co-ordinates of the particle at any time, and (o,o) the co-ordinates of the point of projection, show that $y = -\dfrac{gx^2}{2v^2}$ is the equation of its path, the x-axis being horizontal and the y-axis vertical.
A marble rolls off the top of a stairway horizontally and in a direction perpendicular to the edge of each step, with a speed of 20 feet per second. Each step is 6 in. high and 8 in. wide. Which is the first step struck by the marble (assuming the stairway is sufficiently high) ?

6. Masses of 1 lb., 2 lbs., 3 lbs., and 4 lbs. are placed at points the co-ordinates of which are $(4,1)$, $(-3,2)$, $(2,-3)$, and $(-1,4)$ respectively. Locate the centre of gravity (or the centre of mass) of the system.
Two discs of radii r and $^r\!/_2$ are placed flat on a horizontal plane with a distance $d>2r$ between their centres. If X is their centre of gravity and O the centre of gravity of the larger disc calculate the length of OX.
A square hole is punched out of a circular disc (radius a), a diagonal of the square being a radius of the circle. Show that the centre of gravity of the remainder of the disc is at a distance $\dfrac{a}{4\pi - 2}$ from the centre of the circle.

7. Explain "normal reaction" and "angle of friction".
Deduce $\theta \leqslant \lambda$ where λ is the angle of friction and θ the angle between the normal reaction and the reaction (resultant reaction).
A uniform rod AB 4 feet long leans against the smooth edge of a table three feet in height with one end of the rod on a rough horizontal floor (as in diagram). If the rod is on the point of slipping when inclined at an angle of 60° to the horizontal, find the coefficient of friction.
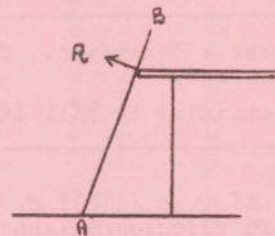
8. Two particles P and Q move with velocities $\vec{v}_1$ and $\vec{v}_2$ respectively. Use diagrams to illustrate clearly the velocity of P relative to Q when $\vec{v}_1$ and $\vec{v}_2$ are

   (i) parallel and in the same direction,
   (ii) parallel and in opposite directions,
   (iii) non-parallel.

A steamer going north-east at 14 knots observes a cruiser which is 10 miles south-east from the steamer and which is travelling in a direction 30° east of north at 25 knots. Illustrate by diagram the cruiser's course as it appears to the steamer. Find the velocity of the cruiser relative to the steamer, in magnitude and direction, and find also the shortest distance between the two ships.

9. Establish an expression for the pressure at a depth d in a liquid of density ϱ at rest under gravity. Hence show that the resultant thrust on a plane surface immersed vertically in the liquid is the product of the area of the plane surface and the depth of its centre of gravity.
Find the resultant horizontal thrust on a rectangular lockgate 20 feet wide when the water stands at a depth of 8 feet on one side and 4 feet on the other.

(1 cu. ft. of water weighs $62\frac{1}{2}$ lbs.).