LEAVING CERTIFICATE EXAMINATION, 1965

APPLIED MATHEMATICS - HONOURS

TUESDAY, 29th JUNE - Morning, 10 to 12.30

Not more than <u>six</u> questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

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1. Two pegs are fixed at points A and D in the same horizontal line. One end of a light string is attached to A and the other end to D. When masses of 5 and 9 lb. are attached to points B and C, respectively, on the string, the $\angle DAB = 30°$ and $\angle ADC = 60°$. Find the tension, in lb. wt., in each of the segments AB, BC, CD of the string, correct in each case to one significant figure.

2. Three particles weighing w_1, w_2, w_3 are placed, respectively, at points the coordinates of which are (x_1, y_1), (x_2, y_2), (x_3, y_3). If $(\bar{x}, \bar{y})$ are the coordinates of the centre of gravity of the system, show that

$$\bar{x} = \frac{x_1 w_1 + x_2 w_2 + x_3 w_3}{w_1 + w_2 + w_3}, \quad \bar{y} = \frac{y_1 w_1 + y_2 w_2 + y_3 w_3}{w_1 + w_2 + w_3}.$$

Particles of weight 3, 1, 2 and 4 grams are placed, respectively, at the points A, B, C, D, the coordinates of which are (2, 5), (2, 1), (4, 1), (6, 8) respectively. Find the position of the centre of gravity of the system.

What additional weight must be placed at B if the centre of gravity is required to be at the mid-point of AC ?

3. What is meant by the velocity of a particle relative to another particle ?

Explain, with the aid of a diagram, how the velocity of A relative to B may be found if the velocities of A and B are known.

The velocity of a ship relative to a steady wind is 20 m.p.h. in the direction 80° North of East and the velocity of a boat relative to the same wind is 10 m.p.h. in the direction 20° South of West. Find the velocity (in magnitude and direction) of the ship relative to the boat.

4. A car weighing 15 cwt. is descending an incline of 1 in 112. The speed of the car is 45 m.p.h. and it is accelerating at the rate of $1\frac{1}{3}$ ft. per sec.^{2}. If the frictional resistance to motion is equivalent to 20 lb. wt., find the horse-power at which the car is working.

If the car travels in a straight line on a horizontal track against a frictional resistance to motion of 50 lb. wt., find the greatest speed it attains if it develops 9 horse-power.

5. If three forces acting at a point are in equilibrium, prove that each force is proportional to the sine of the angle between the lines of action of the other two forces.

The perpendicalars drawn from the vertices of an acute - angled triangle meet at O. The perpendiculars from O to the sides of the triangle are the lines of action of three forces. Each force acts away from O and is proportional to the length of the side to which it is perpendicular. Prove that the three forces are in equilibrium.

6. Derive an expression, in terms of the angle of projection and the initial velocity, for (i) the range, (ii) the greatest height reached by a projectile.

Two particles are projected from a point O with the same initial velocity at angles of elevation α_1 and α_2. If $\alpha_1 + \alpha_2 = 90°$, show that the range of each of the two particles is the same.

If the initial velocity of each of the particles is 48 ft. per second and 30 ft. is the greatest height reached by one of the particles, find the greatest height reached by the other particle.

7. Define simple harmonic motion.

A particle is moving in a straight line with simple harmonic motion. When it is 5 cm. from its mean position, its velocity and acceleration are 5 cm. per sec. and 5 cm. per sec.^{2}, respectively. Find

(i) the greatest velocity of the particle,

(ii) the period of the motion,

(iii) the average velocity of the particle as it travels from rest to rest.

8. A circular piece of tin rotates at the rate of 45 revolutions per minute about an axis through its centre O, the axis being perpendicular to the plane of the tin. A and B are two points on the tin such that OA = 6 inches, OB = 1 ft. 6 ins. and $\angle AOB = 120°$. Weights of 3 lb. and 1 lb. are placed, respectively, at A and B. Find the magnitude and direction of the resultant centrifugal force on the axis.

If C is a point on the tin such that OC = 1 ft., $\angle AOC = 30°$ and $\angle BOC = 150°$, what weight must be placed at C if the resultant centrifugal force is to act along OA ?

9. A vessel in the shape of a cube has internal edges each 2 ft. long. A liquid fills the vessel to a height of 6 inches and a second liquid which does not mix with the first occupies the remainder of the vessel. On each vertical side the total thrust on that part in contact with one liquid is equal to the total thrust on that part in contact with the other. Show that the specific gravities of the two liquids are in the ratio 1 : 3.